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The Sufficient Conditions of Local Controllability for Linear Systems with Random Parameters

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Abstract: This paper is concerned with the problem of local controllability for linear nonstationary systems with random parameters. In differ of well-known problem of controllability for the determinated systems, for systems with random parameters we must construct a non-predicting control when we use the information about system only before the current moment. We obtain the sufficient conditions of non-predicting controllability and estimation of the probability that the given system is a locally controllable on the fixed time segment. The algorithm of construction of the non-predicting control is developed.

Keywords: *Local controllability; non-predicting control; controllability set; stationary stochastic process.*

Mathematics Subject Classification (2000): 93B05, 93B52, 93E20, 37N35, 49J55.

1 Introduction

The problems of controllability, observability and stability of dynamical systems with random parameters was investigated in many works, for example, [1]-[7]. Notice, that for such type of systems we often have not the information about the systems behaviour in future, thats why is appeared a problem of existence of a non-predicting control. The term of the non-predicting control was introduced in Ekaterinburg school on the control theory (see [8, 9]), the problem of such control construction was investigated also in [10, 11]. The control $u(t, x)$ is called the *non-predicting* if for it construction in the moment $t = \tau$ we use the information about system only for $t \leqslant \tau$.

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In this paper we continue the investigation initiated in [12, 13], where we considered the linear systems with the stationary random parameters and obtained the sufficient conditions of existence of the non-predicting control for such systems. In [12, 13] we investigated the conditions of total controllability when we don't assumed any restrictions on the control $u \in \mathbb{R}^m$. Here we consider the system

$$\dot{x} = A(f^t\omega)x + B(f^t\omega)u, \quad (t, \omega, x, u) \in \mathbb{R} \times \Omega \times \mathbb{R}^n \times U, \qquad (1)$$

where the function $t \to \xi(f^t\omega) \doteq \big(A(f^t\omega), B(f^t\omega)\big)$ of variable t is a piecewise constant for every $\omega \in \Omega$. We assume that $u \in U$, where U is a compact convex set in $\mathbb{R}^m$ and U contains the origin in their interior. The aim of this paper is to obtain the sufficient conditions of the non-predicting local controllability for system (1) on the segment $[0, T]$. We prove that in the case $u \in U$ for construction of the non-predicting control we must constantly hold the trajectories of the system (1) solutions in the neighbourhood of the origin, that lead to some additional conditions for the asymptotical behaviour of the system $\dot{x} = A(f^t\omega)x$ solutions.

2 The basic definitions and designations

Suppose $e_1 \doteq \mathrm{col}(1, 0, \ldots, 0), \ldots, e_n \doteq \mathrm{col}(0, \ldots, 0, 1)$ is a standard basis in Euclidean space $\mathbb{R}^n$; $\|x\| = \sqrt{x^*x}$ is a norm in $\mathbb{R}^n$; $\mathrm{Lin}(q_1, \ldots, q_r)$ is a linear hull of the vectors $q_1, \ldots, q_r \in \mathbb{R}^n$; $O^n_\varepsilon(x_0)$ is an ε-neighbourhood of the point x_0 in $\mathbb{R}^n$, $O^n_\varepsilon = O^n_\varepsilon(0)$; $\mathrm{int}\, U$ is an interior of the set U.

Let us consider the probability spaces $(\Omega_1, \mathfrak{F}_1, \mu_1)$ and $(\Omega_2, \mathfrak{F}_2, \mu_2)$, where Ω_1 is a space of number sequences $\theta = (\theta_1, \ldots, \theta_k, \ldots)$, $\theta_k \in (0, \infty)$, the space $\Omega_2 \doteq \{\varphi : \varphi = (\varphi_0, \varphi_1, \ldots \varphi_k, \ldots), \varphi_k \in \Psi\}$, $\Psi = \{\psi_j\}_{j=1}^s$ is a finite set of the matrix pairs $\psi_j = (A_j, B_j)$, $\mathfrak{F}_i$ is a σ-algebra formed by the corresponding cylinder sets, μ_i is an extension of a measure $\widetilde{\mu}_i$ from the algebra of the cylinder sets to the σ-algebra $\mathfrak{F}_i$, $i = 1, 2$. We also consider the probability space $(\Omega, \mathfrak{F}, \mu)$, where $\Omega = \Omega_1 \times \Omega_2$. The construction of σ-algebra $\mathfrak{F}$ and the probability measure μ was described in [2].

On the space $(\Omega_2, \mathfrak{F}_2, \mu_2)$ for every $\theta \in \Omega_1$ we introduce the sequence of random variables $\zeta = (\zeta_0, \zeta_1, \ldots)$ such that $\zeta_k(\omega) = \zeta_k(\varphi, \theta) = \varphi_k$, $\varphi_k \in \Psi$. We suppose that the sequence ζ forms the homogeneous Markov chain, which uniquely determines by the matrix of the transition probabilities $P = (p_{ij})_{i,j=1}^s$ and the initial distribution $\pi = (\pi_i)_{i=1}^s$ (see [14, p. 122]). We also suppose that the Markov chain ζ is a *stationary in the narrow sense* (see [14, p. 432]).

Let us introduce the sequence $\{\tau_k\}_{k=0}^\infty : \tau_0 = 0,\ \tau_k(\theta) = \sum\limits_{i=1}^k \theta_i$, where $\theta \in \Omega_1$. We assume that $\theta_1, \theta_2, \ldots$ are the independent positive random variables and $\theta_2, \theta_3, \ldots$ have the equal distribution $F(t)$, $t \in (0, \infty)$ with the mathematical expectation m_θ. Denote by $\nu(t, \theta)$ a number of points of the sequence $\{\tau_k\}$, which lie left than t, that is

$$\nu(t, \theta) = \max\{k : \tau_k \leqslant t\}, \quad t \geqslant 0.$$

The variable $\nu(t)$ is called a recovery process. We assume that $\nu(t)$ is a stationary recovery process (that is this process have a stationary recovery speed), then the distribution of θ_1 satisfies the equality (see [15, p. 145–147])

$$F_1(t) = \frac{1}{m_\theta} \int\limits_0^t (1 - F(x))\, dx, \quad t > 0. \qquad (2)$$

Let us introduce the shift transformation $f_1^t\theta = (\tau_{\nu+1} - t, \theta_{\nu+2}, \theta_{\nu+3}, \dots)$, $t > 0$ on the probability space $(\Omega_1, \mathfrak{F}_1, \mu_1)$. The transformation f_1^t preserves the measure μ_1, because the sequence $\{\tau_k\}$ forms the stationary recovery process. We also introduce the shift transformation $f_2^t(\theta)\varphi = (\varphi_\nu, \varphi_{\nu+1}, \dots)$ on the space $(\Omega_2, \mathfrak{F}_2, \mu_2)$ for any $\theta \in \Omega_1$. From the stationarity of the Markov chain ζ it follows that the transformation f_2^t preserves the measure μ_2. In [16, p. 190] was proved that the shift transformation $f^t\omega = f^t(\theta, \varphi) = \big(f_1^t\theta, f_2^t(\theta)\varphi\big)$ on the space $(\Omega, \mathfrak{F}, \mu)$ preserves the measure μ.

Assume that $\xi(\omega) = \zeta_0(\omega)$ is a stochastic variable on the probability space $(\Omega, \mathfrak{F}, \mu)$. We introduce the random process $\xi(f^t\omega) = \big(A(f^t\omega), B(f^t\omega)\big)$ generated by the flow $f^t\omega$. Then $\xi(f^t\omega)$ receives the constant values φ_k for $t \in [\tau_k, \tau_{k+1})$. The function $\xi(f^t\omega)$ is a stationary in the narrow sense random process (see [14, p. 433], [16, p. 167], [17, p. 189]). We remind that the process $\xi(t, \omega)$ is called *stationary in the narrow sense* if the equality $\mu(f^tG) = \mu(G)$ satisfies for any cylinder set $G \in \mathfrak{F}$ (see [16, p. 174]).

We identify the system (1) with the function $\xi : \Omega \to \Psi$. For each fixed ω the function $\xi(f^t\omega)$ designates a linear determinate system. We say that *an admissible control* of the system ξ is any bounded and Lebesgue measurable function $u_\omega : \mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^n \to U \in \mathbb{R}^m$. The control type $u_\omega(t, x_0)$ is said to be *program control* if it is not explicitly depends from x; the control type $u_\omega(t, x)$ is said to be *positional control.* The program control $u_\omega(t, x_0)$ is said to be *non-predicting* on the segment $[t_0, t_1]$ if for it construction in the moment $\tau \in [t_0, t_1]$ we use the information about matrices $A(f^t\omega)$ and $B(f^t\omega)$ only for $t \leqslant \tau$ (and not use the information for $t > \tau$).

Let us consider the intervals $[\tau_k, \tau_{k+1})$, where the function $\xi(f^t\omega)$ receives the constant values $\varphi_k \in \Psi$. On any interval $[\tau_k, \tau_{k+1})$ the system ξ coincides with one of the systems ξ_i, $i = 1, \dots, s$, where over ξ_i we denote the system

$$\dot{x} = A_i x + B_i u, \quad (x, u) \in \mathbb{R}^n \times U.$$

Here U is a compact convex set in $\mathbb{R}^m$ and U contains the origin in their interior. In this work we construct the non-predicting control in such form that on any interval $[\tau_k, \tau_{k+1})$, $k = 0, 1, \dots$ we apply either the positional control, or at first the program control for $t \in [\tau_k, \tau_k + \alpha)$, then the positional control for $t \in [\tau_k + \alpha, \tau_{k+1})$. Therefore let us improve in what sense we determine the solution of the system ξ under the fixed $\omega \in \Omega$. We introduce the sequence $\{\vartheta_k\}_{k=0}^\infty$, where $\vartheta_0 = 0$, $\vartheta_{k+1} > \vartheta_k$ such that on the intervals $[\vartheta_k, \vartheta_{k+1})$, $k = 1, \dots$ we apply either only the program control, or only the positional one in dependence from the number of system ξ_i that appeared in the corresponding time moment. If we construct the program control $u_\omega(t)$ on the interval $[\vartheta_k, \vartheta_{k+1})$, then the solution of the system ξ is an absolutely continuous function $x(t) = x(t, \vartheta_k, x_k, u_\omega)$, $x(\vartheta_k) = x_k$, which satisfies the corresponding system $\dot{x} = A_i x + B_i u_\omega(t)$ for almost all $t \in [\vartheta_k, \vartheta_{k+1})$. For the continuity of the solution we require that $x(\vartheta_k, \vartheta_{k-1}, x_{k-1}) = x_k$. Now we assume that on the interval $[\vartheta_k, \vartheta_{k+1})$ we must construct the positional control $u = u_\omega(t, x)$. Let us consider the system ξ_i closed by the control $u = u_\omega(t, x)$ and denote by $x(t) = x(t, \vartheta_k, x_k, u_\omega)$ the solution of this system. We require that $x(t)$ satisfies the conditions $x(\vartheta_k) = x_k$, $x(\vartheta_k, \vartheta_{k-1}, x_{k-1}) = x_k$. Let us denote $u_\omega(t) = u_\omega(t, x(t))$. Then for any initial point x_k the solution of the system $\dot{x} = A_i x + B_i u_\omega(t, x)$ we can also obtain as the solution of the control system ξ_i that corresponds the control $u_\omega(t)$, see [18, p. 431–433].

Definition 2.1 The state $x_0 \in \mathbb{R}^n$ of system $\xi(f^t\omega)$ is said to be *controllable (non-predicting controllable) on the segment* $[t_0, t_1]$ if there exists a control $u_\omega(t, x, x_0)$ (non-predicting control $u(f^t\omega, x, x_0))$, $t \in [t_0, t_1]$ such that the corresponding solution $x(t, \omega)$, $x(t_0, \omega) = x_0$ satisfies $x(t_1, \omega) = 0$.

We denote by $D_{[t_0,t_1]}(\omega)$ the controllability set of the system $\xi(f^t\omega)$ on the segment $[t_0, t_1]$, that is the set of all points, which can be steered to zero on $[t_0, t_1]$ under the fixed $\omega \in \Omega$. We also denote by $\mathcal{D}_{[t_0,t_1]}(\omega)$ the set of all non-predicting controllable states of the system $\xi = \xi(f^t\omega)$ on the segment $[t_0, t_1]$.

Definition 2.2 The system ξ is said to be *locally controllable with the probability* μ_0 on the segment $[t_0, t_1]$ if $\mu\{\omega : 0 \in \operatorname{int} D_{[t_0,t_1]}(\omega)\} = \mu_0$ and *non-predicting locally controllable with the probability* μ_0 on the segment $[t_0, t_1]$ if the probability $\mu\{\omega : 0 \in \operatorname{int} \mathcal{D}_{[t_0,t_1]}(\omega)\} = \mu_0$.

3 The Construction of the Positional Control

Let us consider the system ξ_i and denote by K_i the matrix

$$K_i = (B_i, A_i B_i, \dots, A_i^{n-1} B_i), \quad i = 1, \dots, s,$$

by $D_{[t_0,t_1]}(\xi_i)$ the controllability set of the system ξ_i on the segment $[t_0, t_1]$, by $L(\xi_i) \doteq \operatorname{Lin} D_{[t_0,t_1]}(\xi_i)$ the controllability space of the system ξ_i, by $X_i(t, s) = X_i(t - s)$ the Cauchy matrix of this system. It is known that the controllability space $L(\xi_i)$ coincides with the subspace formed by the columns of the matrix K_i, that is $L(\xi_i) = \operatorname{Lin} K_i$. Therefore the condition $\operatorname{rank} K_i = n$ is the necessary and sufficient condition of the local controllability for system ξ_i (see [19, p. 140–145]).

Let us consider a determinate system ξ_0, which coincides with the system ξ_{i_ℓ} on any interval $[(\ell - 1)\alpha, \ell\alpha)$, $\ell = 1, \dots, k$, that is $\xi_0 = \psi_{i_\ell}$ for $t \in [(\ell - 1)\alpha, \ell\alpha)$. We can consider the system ξ_0 as the system ξ under the fixed $\omega = (\theta, \varphi)$ with k first coordinates $\omega_\ell = (\alpha, \psi_{i_\ell})$.

Lemma 3.1 [20] *Assume that $\xi_0 = \psi_{i_\ell}$ for $t \in [(\ell - 1)\alpha, \ell\alpha)$, $\ell = 1, \dots, k$. Then the controllability space of system ξ_0 on the segment $[(\ell - 1)\alpha, k\alpha]$*

$$L_{[(\ell-1)\alpha,k\alpha]}(\xi_0) = L(\xi_{i_\ell}) + X_{i_\ell}^{-1}(\alpha) L(\xi_{i_{\ell+1}}) + \dots + X_{i_\ell}^{-1}(\alpha) \cdot \ldots \cdot X_{i_{k-1}}^{-1}(\alpha) L(\xi_{i_k}).$$

Suppose that for system ξ there exists $\omega \in \Omega$ such that the corresponding determinate system ξ_0 is a totally controllable on the segment $[0, k\alpha]$, that is the controllability space $L_{[0,k\alpha]}(\xi_0)$ coincides with $\mathbb{R}^n$. In the present work we investigate the next problem: is it possible to construct the non-predicting control for the system ξ and what is the probability that this system is the non-predicting controllable on the fixed time segment $[0, T]$ (in the process of construction of such control we assume that for the system ξ in the moment τ the moments of switching τ_k and the states of this system for $t > \tau$ are unknown). Further we propose the algorithm of construction of the non-predicting control when it is not sufficient the equality $L_{[0,k\alpha]}(\xi_0) = \mathbb{R}^n$. The subspaces $L_{[(\ell-1)\alpha,k\alpha]}(\xi_0)$, $\ell = 2, \dots, k$ must satisfy some additional condition, that is the trajectory of the system under some control must retains in given subspace to the next moment of switching. In Lemma 3.2 we obtain the condition of such retaining when there are not any restrictions on the control.

Lemma 3.2 [12] *Let $\mathcal{M}$ be a subspace in $\mathbb{R}^n$ and M be a matrix formed from the vectors of basis $\mathcal{M}$. If for the system*

$$\dot{x} = Ax + Bu, \quad (x,u) \in \mathbb{R}^n \times \mathbb{R}^m, \tag{3}$$

we have $\mathrm{Lin}\, AM \subset \mathrm{Lin}(M,B)$, then there exists a positional control $u(x)$ such that for any point $x_0 \in \mathcal{M}$ the trajectory of solution $x(t,t_0,x_0,u)$ contains in the subspace $\mathcal{M}$ for all $t \geqslant 0$.

Further we consider the system $S : \dot{x} = Ax + Bu$, $(x,u) \in \mathbb{R}^n \times U$, where $U \subset \mathbb{R}^m$ is a compact convex set containing the origin in their interior. We denote by $L(S) \doteq \mathrm{Lin}\, D_{[t_0,t_1]}(S)$, then $L(S)$ is a controllability space for the system (3).

In the next statement we obtain the sufficient conditions of existence of the positional control $u(x) \in U$ for the system S. This control must retains the trajectory of solution $x(t,t_0,x_0,u)$ on the subspace $\mathcal{M}$ for $t \geqslant t_0$, if x_0 is a point located on this subspace in the moment t_0. Furthermore, for $u(x) \in U$ must exists $\varepsilon > 0$ such that from the inequality $\|x_0\| < \varepsilon$ follows that the solution $\|x(t,t_0,x_0,u)\| < \varepsilon$ for all $t \geqslant t_0$.

We denote by $\lambda_1, \dots, \lambda_p$ the eigenvalues of matrix A corresponding to the different Jordan cells (for this eigenvalues not required to be different), by m_k we denote the size of Jordan cell corresponding to the eigenvalue λ_k. We also denote by Λ the set of eigenvalues λ_k such that either $\mathrm{Re}\lambda_k > 0$ or $\mathrm{Re}\lambda_k = 0$ and the size of corresponding Jordan cell is more than one, that is

$$\Lambda = \{\lambda_k : \lambda_k \in \{\mathrm{Re}\lambda_k > 0\} \cup \{\mathrm{Re}\lambda_k = 0, m_k > 1\}\}.$$

Lemma 3.3 *Let $\mathcal{M}$ be a subspace in $\mathbb{R}^n$ and M be a matrix from the vectors of basis $\mathcal{M}$. Suppose that the system S and the subspace $\mathcal{M}$ satisfy the conditions:*
(1) $\mathcal{M} \cap L(S) = \{0\}$;
(2) $\mathrm{Lin}\, AM \subset \mathrm{Lin}(M,B)$;
(3) *the controllability space $L(S)$ contains all rooted subspaces of matrix A, corresponding to the eigenvalues $\lambda_k \in \Lambda$.*
Then there exists the positional control $u(x) \in U$, for which we can find $\varepsilon > 0$ and $\delta = \delta(\varepsilon) > 0$ such that for any point $x_0 \in \mathcal{M} \cap O_\delta$ the trajectory of solution $x(t,t_0,x_0,u)$ contains in $\mathcal{M} \cap O_\varepsilon$ for all $t \geqslant t_0$.

Proof Assume that for the system S a dimension of the controllability space $\dim L(S) = r$. Then there exists a linear transformation $x = Cy$ that reduce the system S to the system type $\widetilde{S} = (\widetilde{A}, \widetilde{B})$:

$$\dot{y}^1 = A_{11}y^1 + A_{12}y^2 + B_1\widetilde{u},$$

$$\dot{y}^2 = A_{22}y^2,$$

where $y^1 \in \mathbb{R}^r$, $y^2 \in \mathbb{R}^{n-r}$ and the controllability subspace $L(\widetilde{S})$ determines in $\mathbb{R}^n$ by the equation $y^2 = 0$ (see [18, p. 110]). From the equalities $\widetilde{A} = C^{-1}AC$ and $\widetilde{B} = C^{-1}B$ it is easy to verify that the controllability spaces of the systems S and $\widetilde{S}$ satisfy the condition

$$L(\widetilde{S}) = C^{-1}L(S). \tag{4}$$

Let us denote $\widetilde{\mathcal{M}} = C^{-1}\mathcal{M}$. Then from the conditions (1) and (4) follows that $\widetilde{\mathcal{M}} \cap L(\widetilde{S}) = \{0\}$. The conditions (2) and $\mathrm{Lin}\, C\widetilde{A}\widetilde{M} \subset \mathrm{Lin}(C\widetilde{M}, C\widetilde{B})$ are equivalent, therefore $\mathrm{Lin}\, \widetilde{A}\widetilde{M} \subset \mathrm{Lin}(\widetilde{M}, \widetilde{B})$.

It is known that the similar matrices A and $\widetilde{A}$ have the equal eigenvalues λ_k, $k = 1, \ldots, p$. Let ℓ_i and $\widetilde{\ell}_i$ be the eigen and the adjoint vectors of matrices A and $\widetilde{A}$. If ℓ_i and $\widetilde{\ell}_i$ correspond to the equal λ_i, then we have $\ell_i = C\widetilde{\ell}_i$ (see [21, p. 31]). Therefore the condition (3) is equivalent the follow condition: the subspace $L(\widetilde{S})$ contains all rooted subspaces of matrix $\widetilde{A}$, corresponding to the $\lambda_k \in \Lambda$.

Note, that the vectors $\widetilde{\ell}_i$ type $\widetilde{\ell}_i = (\ell_i^1, 0)$, $\ell_i^1 \in \mathbb{R}^r$, $i = 1, \ldots, r$, are contained in the controllability subspace $L(\widetilde{S}) = \mathrm{Lin}(e_1, \ldots, e_r)$. The matrix $\widetilde{A}$ also have the rooted subspaces formed by the vectors $\widetilde{\ell}_i = (\ell_i^1, \ell_i^2)$, $\ell_i^2 \neq 0$, $i = r+1, \ldots, n$ that dont lie in $L(\widetilde{S})$. Here $\ell_i^1 \in \mathbb{R}^r$, $\ell_i^2 \in \mathbb{R}^{n-r}$ and vectors ℓ_i^2 are the eigen or the adjoint ones of matrix A_{22} (vectors $\widetilde{\ell}_i$ and ℓ_i^2, $i = r+1, \ldots, n$ correspond to the equal eigenvalues). Since (3), it follows that for these eigenvalues $\mathrm{Re}\lambda_k < 0$ or $\mathrm{Re}\lambda_k = 0$ and $m_k = 1$.

Using the conditions $L(\widetilde{S}) = \mathrm{Lin}(e_1, \ldots, e_r)$ and $\widetilde{\mathcal{M}} \cap L(\widetilde{S}) = \{0\}$, we get that the subspace $\widetilde{\mathcal{M}}$ don't contains the unit vectors $e_1, \ldots, e_r$. Therefore we can represent this subspace in the form

$$\widetilde{\mathcal{M}} = \mathrm{col}(\widetilde{\mathcal{M}}_1, \widetilde{\mathcal{M}}_2) = \mathrm{Lin}(h_1, \ldots, h_j), \quad h_i = \mathrm{col}(h_i^1, h_i^2),$$

$$\widetilde{\mathcal{M}}_1 = \mathrm{Lin}(h_1^1, \ldots, h_j^1), \quad \widetilde{\mathcal{M}}_2 = \mathrm{Lin}(h_1^2, \ldots, h_j^2), \quad j \leqslant n - r,$$

where vectors $h_i^1 \in \mathbb{R}^r$, h_i^2 are the linear independent vectors in $\mathbb{R}^{n-r}$.

We denote by $y(t) = y(t, t_0, y_0, \widetilde{u}) = \mathrm{col}(y^1(t), y^2(t))$ the solution of the system $\widetilde{S}$ closed by the control $\widetilde{u}(y) \in U$. Here $y^1(t) = y^1(t, t_0, y_0^1, \widetilde{u})$ and $y^2(t) = y^2(t, t_0, y_0^2)$ is the solution of the system $\dot{y}^2 = A_{22}y^2$. Let us obtain the solution $y(t)$ such that its trajectory, going in the moment t_0 from the point $y_0 = (y_0^1, y_0^2) \in \widetilde{\mathcal{M}}$, remains in the subspace $\widetilde{\mathcal{M}}$ for all $t \geqslant t_0$. Note, that from the condition $\mathrm{Lin}\,\widetilde{A}\widetilde{M} \subset \mathrm{Lin}(\widetilde{M}, \widetilde{B})$ follows the condition $\mathrm{Lin}\,A_{22}\widetilde{M}_2 \subset \mathrm{Lin}\,\widetilde{M}_2$, which means that for every point $y_0^2 \in \widetilde{\mathcal{M}}_2$ the trajectory of $y^2(t)$ contains in the subspace $\widetilde{\mathcal{M}}_2 = \mathrm{Lin}(h_1^2, \ldots, h_j^2)$ for all $t \geqslant t_0$. Therefore we can represent the solution $y^2(t)$ in the form

$$y^2(t) = \alpha_1(t)h_1^2 + \cdots + \alpha_j(t)h_j^2, \quad \alpha_i(t) = \sum_{l=1}^{q} e^{\lambda_l t} Q_{il}(t),$$

where the degree of polynomials $Q_{il}(t)$ not more than $m_l - 1$. The solution $y^2(t)$ is bounded for $t_0 \leqslant t < \infty$ because the eigenvalues λ_k of matrix A_{22} satisfy the condition $\mathrm{Re}\lambda_k < 0$ or $\mathrm{Re}\lambda_k = 0$ and $m_k = 1$.

Notice, that if $\mathrm{Lin}\,\widetilde{A}\widetilde{M} \subset \mathrm{Lin}(\widetilde{M}, \widetilde{B})$, then for any basic vector $h_i \in \widetilde{\mathcal{M}}$, $i = 1, \ldots, j$ there exists a vector $u_i \in \mathbb{R}^m$ such that $\widetilde{A}h_i + \widetilde{B}u_i \in \widetilde{\mathcal{M}}$. This means that there exists a vector $c_i = \mathrm{col}(c_{1i} \ldots c_{ji})$ such that the system $\widetilde{M}c_i - \widetilde{B}u_i = \widetilde{A}h_i$ has the solution. Let us construct the positional control $\widetilde{u} = \alpha_1(t)u_1 + \ldots + \alpha_j(t)u_j$ and denote by $c = \alpha_1(t)c_1 + \ldots + \alpha_j(t)c_j$. Suppose that $y_0 \in \widetilde{\mathcal{M}}$ and $y(t) = y(t, t_0, y_0, \widetilde{u}) = \alpha_1(t)h_1 + \ldots + \alpha_j(t)h_j$ is the solution of the system $\widetilde{S}$ such that its trajectory lies in the subspace $\widetilde{\mathcal{M}}$. Then the vector $\mathrm{col}(c, \widetilde{u}) \in \mathbb{R}^{k+m}$ is the solution of the system

$$\widetilde{M}c - \widetilde{B}\widetilde{u} = \widetilde{A}y, \quad y \in \widetilde{\mathcal{M}}. \tag{5}$$

Combining $\widetilde{\mathcal{M}} = \mathrm{col}(\widetilde{\mathcal{M}}_1, \widetilde{\mathcal{M}}_2)$, $\widetilde{B} = \mathrm{col}(\widetilde{B}_1, 0)$, $\mathrm{rank}\,\widetilde{M}_2 = j$, $\mathrm{rank}\,\widetilde{B}_1 = m$ and condition (2), we obtain that $\mathrm{rank}(\widetilde{M}, \widetilde{B}) = \mathrm{rank}(\widetilde{M}, \widetilde{B}, \widetilde{A}\widetilde{M}) = j + m$. This implies that the system (5) is compatible and has a unique solution $\widetilde{u}$, which we can represent in the form $\widetilde{u} = \widetilde{u}(y) = Dy$. Here D is some matrix sizes $m \times n$. Then there exists $\varepsilon > 0$ such that $\widetilde{u}(y) \in U$ for $\|y\| < \varepsilon$.

Since we can express the solution $y(t)$ over the same functions $\alpha_i(t)$, $i = 1, \dots, j$ that enters in $y^2(t)$, therefore this solution is also bounded for $t \geqslant t_0$. This means that for any $\varepsilon > 0$ there exists $\delta > 0$ such that $\|y(t)\| < \varepsilon$ for all $t \geqslant t_0$ if $\|y_0\| < \delta$. Let us consider the phase trajectories of the system $\widetilde{S}$ closed by the control $\widetilde{u}(y) \in U$. Thus, we proved that if these trajectories go from the points $y_0 \in \widetilde{\mathcal{M}} \cap O_\delta$, then they lie in the set $\widetilde{\mathcal{M}} \cap O_\varepsilon$ for all $t \geqslant t_0$. Let us put $u(x) = \widetilde{u}(y)$, then the last condition is equivalent the next one: the phase trajectories of the system S, going from the points $x_0 \in \mathcal{M} \cap O_\delta$ under the control $u(x) \in U$, lie in the set $\mathcal{M} \cap O_\varepsilon$ for all $t \geqslant t_0$ (here ε and δ may be other).

In addition, note that the vector $\mathrm{col}(c, u)$ is the solution of the system

$$Mc - Bu = Ax, \quad x \in \mathcal{M}, \tag{6}$$

which is equivalent the system (5). Thus, the lemma is proved. □

Lemma 3.4 *Suppose $L(S)$ contains all rooted subspaces of matrix A, which correspond to the eigenvalues $\lambda_k \in \Lambda_k$. Then there exists the positional control $u(x) \in U$ type $u = Hx$, and for this control there exist $\varepsilon > 0$ and $\delta = \delta(\varepsilon) > 0$ such that for any point $\|x_0\| < \delta$ the solution $\|x(t, t_0, x_0, u\| < \varepsilon$ for all $t \geqslant t_0$.*

Proof Assume that $\dim L(S) = r$. Let us reduce the system S to the system $\widetilde{S}$ by the linear transformation $x = Cy$. The matrix A_{22} of the system $\dot{y}^2 = A_{22}y^2$ have the eigenvalues λ_k such that $\mathrm{Re}\lambda_k < 0$ or $\mathrm{Re}\lambda_k = 0$ and $m_k = 1$. In [22, p. 30] was proved that there exists a control $u = Hx$ that gives to the matrix $A+BH$ of the closed system r predesigned eigenvalues and the rest eigenvalues of $A+BH$ coincide with the eigenvalues of matrix A_{22}. Therefore, we can choose the control $u = Hx$ such that all eigenvalues of matrix $A + BH$ satisfy the condition $\mathrm{Re}\lambda_k < 0$ or $\mathrm{Re}\lambda_k = 0$ and $m_k = 1$. Then there exists $\varepsilon > 0$ that $u(x) \in U$ for $\|x\| < \varepsilon$ and there exists $\delta = \delta(\varepsilon) > 0$ that for any point $\|x_0\| < \delta$ the solution $x(t) = x(t, t_0, x_0, u)$ satisfies the equality $\|x(t, x_0, u(\cdot))\| < \varepsilon$ for all $t \geqslant t_0$. The lemma is proved. □

4 The Conditions of the Non-predicting Local Controllability

We say that the finite sequence $V = (\psi_{i_1}, \dots, \psi_{i_k})$, where $\psi_{i_j} \in \Psi$ is called a word V. Let us put in correspondence to the word V the linear systems $\xi_{i_1}, \dots, \xi_{i_k}$, the controllability spaces of these systems $L(\xi_{i_1}), \dots, L(\xi_{i_k})$ and the controllability spaces $L_{[(\ell-1)\alpha, k\alpha]}(\xi_0)$, $\ell = 1, \dots, k$, constructed in Lemma 3.1.

Let us denote by $\mu(T)$ the probability of appearance the word V on the segment $[0, T]$.

Lemma 4.1 *Suppose that $0 < \alpha \leqslant \theta_k \leqslant \beta$ for all $k = 2, \dots,$ the set $\Psi = \{\psi_1, \psi_2\}$, the word $V = (\psi_{i_1}, \psi_{i_2})$. Then for $T \geqslant 2N\beta$, $N = 1, 2, \dots,$ the probability $\mu(T)$ satisfies the inequality*

$$\mu(T) \geqslant (1 - \pi_{i_2} p_{i_2 i_2}^{N-1})(1 - p_{i_1 i_1}^{N}). \tag{7}$$

Proof Here we consider the case, when the set Ψ contains two states, then the probability $\mu(T)$ equals to the probability of appearance the word $V = (\psi_{i_1}, \psi_{i_2})$ on the segment $[0, T]$. Notice that $\mu(T)$ not less than the probability of transition the system from any initial state to the state ψ_{i_1} over not more than N steps and then from ψ_{i_1} to ψ_{i_2} also not more than for N steps. It is clear that for such transition of the system on the segment $[0, T]$ must appeared not less than $2N$ jumps of the process, that always true for $T \geqslant 2N\beta$. Let us denote by $f_{i_1i_1}(N)$ the conditional probability of the first reaching of the system the state ψ_{i_1} from this own initial state not more than over N steps. The probability $f_{i_1i_1}(N)$ equals to the probability that the system either reaches the state ψ_{i_1} for one step or goes to ψ_{i_2}, then a few times goes again to ψ_{i_2} and then reaches the initial state ψ_{i_1}, hence

$$f_{i_1i_1}(N) = p_{i_1i_1} + p_{i_1i_2}p_{i_2i_1}(1 + p_{i_2i_2} + \ldots + p_{i_2i_2}^{N-2}) = 1 - p_{i_1i_2}p_{i_2i_2}^{N-1}.$$

Let $f_{i_2i_1}(N)$ be the conditional probability of the first reaching of the system the state ψ_{i_1} from the state ψ_{i_2} not more than over N steps. For this aim the system from the state ψ_{i_2} can reach the state ψ_{i_1} either over one step or at first it can go a few times to ψ_{i_2}, then it goes to ψ_{i_1}, therefore,

$$f_{i_2i_1}(N) = p_{i_2i_1}(1 + p_{i_2i_2} + \ldots + p_{i_2i_2}^{N-1}) = 1 - p_{i_2i_2}^{N}.$$

In the same way, we denote the probability $f_{i_1i_2}(N)$, then $f_{i_1i_2}(N) = 1 - p_{i_1i_1}^{N}$. Further note that the system can reach the state ψ_{i_1} either from ψ_{i_1} or from ψ_{i_2}, hence for $T \geqslant 2N\beta$ we have the inequality

$$\mu(T) \geqslant \Big(\pi_{i_1} f_{i_1i_1}(N) + \pi_{i_2} f_{i_2i_1}(N)\Big) f_{i_1i_2}(N) = \Big(1 - \pi_{i_1}p_{i_1i_2}p_{i_2i_2}^{N-1} - \pi_{i_2}p_{i_2i_2}^{N}\Big)(1 - p_{i_1i_1}^{N}).$$

It is well known that if the Markov chain is a stationary in the narrow sense, then the initial and transition probabilities satisfy the equations $\sum_{j=1}^{s} \pi_j p_{jk} = \pi_k$, $k = 1, \ldots, s$. Hence in the case $s = 2$ we have $\pi_{i_1}p_{i_1i_2} + \pi_{i_2}p_{i_2i_2} = \pi_{i_2}$. Therefore $\mu(T) \geqslant (1 - \pi_{i_2}p_{i_2i_2}^{N-1})(1 - p_{i_1i_1}^{N})$. Thus, the lemma is proved. □

Let $p_{ij}^{(\ell)}$ be the probability of transition from the state ψ_i to the state ψ_j over ℓ steps. The state ψ_j is called an *attainable* from the state ψ_i if there exists $\ell \geqslant 0$ such that $p_{ij}^{(\ell)} > 0$. The states ψ_i and ψ_j are called the *connected* if the state ψ_j is attainable from the state ψ_i and the state ψ_i is attainable from ψ_j (see [14, p. 598]).

Theorem 4.1 *Suppose that for the system ξ the set $\Psi = \{\psi_1, \psi_2\}$, the states ψ_1, ψ_2 are connected and $0 < \alpha \leqslant \theta_k \leqslant \beta$ for all $k = 2, \ldots$ If there exist a word $V = (\psi_{i_1}, \psi_{i_2})$ and a subspace $\mathcal{M} \subset L(\xi_{i_2})$ such that:*

(1) $\mathcal{M} \cap L(\xi_{i_1}) = \{0\}$, $L(\xi_{i_1}) + \mathcal{M} = \mathbb{R}^n$;

(2) $\text{Lin}A_{i_1}M \subset \text{Lin}(M, B_{i_1})$;

(3) *the controllability space $L(\xi_{i_1})$ contains all rooted subspaces of matrix A_{i_1} and the controllability space $L(\xi_{i_2})$ contains all rooted subspaces of A_{i_2}, corresponding to the eigenvalues $\lambda_k \in \Lambda$,*

then the system ξ is non-predicting controlled on $[0, T]$ with probability $\mu(T)$ that satisfies (7) for all $T \geqslant 2N\beta$, $N = 1, 2, \ldots$.

The probability $\mu(T) \to 1$ as $T \to \infty$.

Proof Let us describe the construction of the non-predicting control for the system ξ that satisfies the conditions of the theorem.

1. First let us consider the case, when in the initial moment the system ξ is in the state ψ_{i_1}. The first task is to translate the points $x_0 \in O_\varepsilon$ to the set $\mathcal{M} \cap O_{\varepsilon_1}$ by the program control $u(t) \in U$ for time α. We denote by $D_{[t_0,t_1]}(S, M_0)$ the *controllability set of the system S to the set M_0* on the segment $[t_0, t_1]$. The point x_0 lies in the set $D_{[t_0,t_1]}(S, M_0)$ if and only if there exists an admissible control $u(t)$ such that the solution $x(t) = x(t, t_0, x_0, u)$ of the system S satisfies the condition $x(t_1) \in M_0$. It is known that the set $D_{[t_0,t_1]}(S, M_0)$ satisfies the equality

$$D_{[t_0,t_1]}(S, M_0) = D_{[t_0,t_1]}(S) + X^{-1}(t_1 - t_0)M_0.$$

Here under the algebraic sum of the sets A and B from $\mathbb{R}^n$ we intend the set $A + B = \{a + b : a \in A, b \in B\}$, by $X(t,s) = X(t-s)$ we denote the Cauchy matrix of the system $\dot{x} = Ax$. We obtain

$$\mathrm{Lin}D_{[0,\alpha]}(\xi_{i_1}, \mathcal{M} \cap O_{\varepsilon_1}) = \mathrm{Lin}\Big(D_{[0,\alpha]}(\xi_{i_1}) + X_{i_1}^{-1}(\alpha)(\mathcal{M} \cap O_{\varepsilon_1})\Big) =$$

$$= L(\xi_{i_1}) + X_{i_1}^{-1}(\alpha)\mathcal{M}.$$

In the work [20] was proved that the conditions $L(\xi_{i_1}) + X_{i_1}^{-1}(\alpha)\mathcal{M} = \mathbb{R}^n$ and $L(\xi_{i_1}) + \mathcal{M} = \mathbb{R}^n$ are equivalent, hence from the condition (1) it follows that $\mathrm{Lin}D_{[0,\alpha]}(\xi_{i_1}, \mathcal{M} \cap O_{\varepsilon_1}) = \mathbb{R}^n$. Since $\{0\} \in \mathrm{int}\mathcal{M}$ and $\{0\} \in \mathrm{int}D_{[0,\alpha]}(\xi_{i_1})$, then $\{0\} \in \mathrm{int}D_{[0,\alpha]}(\xi_{i_1}, \mathcal{M} \cap O_{\varepsilon_1})$. Therefore the set $D_{[0,\alpha]}(\xi_{i_1}, \mathcal{M} \cap O_{\varepsilon_1})$ contains some neighbourhood O_ε of the origin such that all points of O_ε reach the set $\mathcal{M} \cap O_{\varepsilon_1}$ by $u(t) \in U$ for time α.

Let us suppose that the system ξ have not the jumps for time $t = \alpha$, that is $\tau_1 \geqslant \alpha$. Since the system ξ_{i_1} and the subspace $\mathcal{M}$ satisfy the conditions of lemma 3.3, then there exists the positional control $u(x) \in U$, which retains the solution $x(t) = x(t, \alpha, x_\alpha, u)$, $x(\alpha) = x_\alpha$ on the subspace $\mathcal{M}$ for all $t \geqslant \alpha$. In this case for every $\varepsilon_2 > 0$ there exists $\varepsilon_1 > 0$ such that for all $\|x_0\| < \varepsilon_1$ the solution $\|x(t)\| < \varepsilon_2$ for all $t \geqslant \alpha$. Suppose that in the moment τ_1 the state ψ_{i_2} is appeared; then we can translate the points of $\mathcal{M} \cap O_{\varepsilon_2}$ to null for time α, because $\mathcal{M}$ contains in the controllability set $L(\xi_{i_2})$. In this case we choose ε_2 such that the program control $u(t) \in U$ for $t \in [\tau_1, \tau_1 + \alpha]$. The case $\tau_1 < \alpha$ considered further in item 3.

2. Suppose that in the initial moment the system ξ is in the state ψ_{i_2}. In this case we must wait for the moment of jump τ_1 during some unknown time and simultaneously choose the control $u(x) \in U$ that satisfy the follow condition: there exist $\varepsilon > 0$ and $\delta = \delta(\varepsilon) > 0$ that all points from the neighbourhood O_δ contain in O_ε for any long time (to the moment τ_1). In Lemma 3.4 we prove the existence of such control $u(x) \in U$. If the state ψ_{i_2} appears again in the next moments of jumping $\tau_1, \dots, \tau_k$, then we keep on restrain the trajectory of the system in the neighbourhood O_ε until the state ψ_{i_1} appeared in some moment τ_{k+1}. For $t \geqslant \tau_{k+1}$ we construct the control as in item 1.

3. Notice that in the initial moment we don't know about the time τ_1 of first jump of the process, thats why we cannot always reach the sets constructed above for this time. Therefore for $t < \tau_1$ we must construct the program control similarly as in the first or second item in dependence of the state of the system in the initial moment. Now suppose that the first jump of the process was in the moment $\tau_1 < \alpha$ and we don't reach the necessary sets for this time, then after the moment τ_1 we have a reserve time α without the next moment of jump τ_2 (because $\theta_k \in [\alpha, \beta]$). Thus, for $t \geqslant \tau_1$ we build the control as above in dependence from the number of state in the moment τ_1.

4. Finally let us prove that $\mu(T) \to 1$ as $T \to \infty$. The states ψ_1, ψ_2 are connected, hence $p_{11} \neq 1$, $p_{22} \neq 1$. Therefore from the inequality (7) we have that $\mu(T) \to 1$ as

$N \to \infty$. Notice, that in this case $T \to \infty$, because $T > \alpha(N-1)$, $\alpha > 0$. Thus, the theorem is proved. □

5 Illustrative Example

Assume that the system ξ has two states $\psi_1 = (A_1, B_1)$, $\psi_2 = (A_2, B_2)$ with the next matrices:

$$A_1 = \begin{pmatrix} 2 & 0 & 0 \\ 1 & 1 & 2 \\ 0 & 0 & -1 \end{pmatrix}, B_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}, A_2 = \begin{pmatrix} -1 & 1 & 0 \\ 0 & -1 & 4 \\ 1 & 1 & -2 \end{pmatrix}, B_2 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}.$$

It is also given the matrix of transition probabilities $P = \begin{pmatrix} 3/5 & 2/5 \\ 4/5 & 1/5 \end{pmatrix}$ and the initial distribution $\pi = (2/3, 1/3)$. Note, that the initial and transition probabilities satisfy the equations $\sum_{j=1}^{2} \pi_j p_{ji} = \pi_i$, $i = 1, 2$. We also suppose that the length of intervals between the system jumps $\theta_k \in [0,5; 1]$, $k = 2, 3 \ldots$, then from (2) follows that $\theta_1 \in [0; 1]$.

It is easily shown that the controllability spaces of these systems $L(\xi_1) = \operatorname{Lin} B_1$, $L(\xi_2) = \operatorname{Lin} B_2$. We choose the word $V = (\psi_1, \psi_2)$ and the subspace $\mathcal{M} = L(\xi_2)$, then the subspaces $\mathcal{M}$ and $L(\xi_1)$ satisfy the equalities:

$$\mathcal{M} \cap L(\xi_1) = \{0\}, \quad L(\xi_1) + \mathcal{M} = \mathbb{R}^3, \quad \operatorname{Lin} A_1 \mathcal{M} \subset \operatorname{Lin}(\mathcal{M}, B_1) = \mathbb{R}^3.$$

Further, the controllability space $L(\xi_1)$ contains the eigenvectors of matrix A_1, $v_1 = \operatorname{col}(0,1,0)$ and $v_2 = \operatorname{col}(1,1,0)$ that correspond to the eigenvalues $\lambda_1 = 1$ and $\lambda_2 = 2$; the matrix A_1 also has the eigenvalue $\lambda_3 = -1$. The subspace $L(\xi_2)$ contains the eigenvector $v_1 = \operatorname{col}(1,2,1)$ of matrix A_2, corresponding $\lambda_1 = 1$, the other eigenvalues of A_2 are $\lambda_2 = -2$, $\lambda_3 = -3$. From the Theorem 4.1 it follows that the system ξ is the non-predicting controlled on the segment $[0, T]$ with the probability $\mu(T)$, which satisfies the next inequality for $T \geqslant 2N$:

$$\mu(T) \geqslant \left(1 - \frac{1}{3} \cdot 0,2^{N-1}\right)(1 - 0,6^N).$$

Let us describe the construction of the non-predicting control for this system and obtain the corresponding positional controls. Assume that in the initial moment the system ξ is in the state ψ_1. First we translate the points $x_0 \in O_\varepsilon$ to the set $\mathcal{M} \cap O_{\varepsilon_1}$ by the program control $u(t) \in U$ for the time $\alpha = 0,5$. If the system has not the jumps during the time interval α, that is $\tau_1 \geqslant \alpha$, then we restrain the trajectories of the system ξ_1 in the set $\mathcal{M} \cap O_{\varepsilon_1}$ by the control $u(x)$ to the jump moment τ_k, when the system goes to the state ψ_2. For obtaining the control $u(x)$ we represent the vector $x \in \mathcal{M}$ in the form $x = \operatorname{col}(x_1, 2x_1, x_1)$, then from the system (6) we have $u(x) = \operatorname{col}(u_1, u_2) = \operatorname{col}(-3x_1, -7x_1)$. We obtain the solution $x(t, \alpha, x_0, u)$ of the system ξ_1, closed by the control $u(x)$, going from the point $x_0 = (x_0^1, 2x_0^1, x_0^1)$:

$$x(t, \alpha, x_0, u) = \operatorname{col}\left(x_1^0 e^{-(t-\alpha)}, 2x_1^0 e^{-(t-\alpha)}, x_1^0 e^{-(t-\alpha)}\right).$$

Note, that this solutions satisfies the inequality $\|x(t, \alpha, x_0, u)\| \leqslant \|x_0\| < \varepsilon_1$ and its trajectory contains in the subspace $\mathcal{M}$ for all $t \geqslant \alpha$. Further, when the state ξ_2 appears in the moment τ_k, we translated the points from $\mathcal{M} \cap O_{\varepsilon_1}$ to null by the corresponding program control.

Suppose that $\tau_1 < \alpha$ and in the moment τ_1 the state ψ_2 appears, then the trajectories of the system cannot always reach the set $\mathcal{M} \cap O_{\varepsilon_1}$ for the moment τ_1. In this case after τ_1 we must restrain the trajectories in some neighbourhood of the origin for the moment τ_q, when the system will be in the state ψ_1 again. For this aim we construct the positional control for the system $\xi_2 : u(x) = -x_1 - x_2$, such that all eigenvalues of the matrix of closed system are equal -2. Then there exist $\varepsilon > 0$ that $u(x) \in U$ for $\|x\| < \varepsilon$ and $\delta = \delta(\varepsilon) > 0$ that for any point $\|x_0\| < \delta$ the solution $\|x(t, \tau_1, x_0, u)\| < \varepsilon$ for all $t \geqslant \tau_1$. After appearing the state ψ_1 we deal as in the first case. In the same way, if in the moment $t = 0$ appears the state ψ_2, we must restrain the trajectories in some neighbourhood of the origin for the moment τ_q, when the system will be in the state ψ_1.

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