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(1908 – 1988)

ABSTRACTS

МЕЖДУНАРОДНАЯ КОНФЕРЕНЦИЯ

ДИФФЕРЕНЦИАЛЬНЫЕ УРАВНЕНИЯ И ТОПОЛОГИЯ

посвященная 100-летию со дня рождения Льва Семёновича Понтрягина
(1908 – 1988)

ТЕЗИСЫ ДОКЛАДОВ

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where $X = (r, v)$ and by $F(t,X,U)$ the convex envelope of the set of vectors $\eta(t, X, U)$, $u \in U$. Let some payoff functional $J(t, f, u)$ be given.

Main result of the communication consist in the following. *To begin with, the find some smooth potential [2] in the form of a solution of specially -constructed Fredholm integral equation of second order. Then, with its help and in accordance with the receipts of the positional differential games [2, p.106-113] we develope an algorithm for a synthesis of the exterior charged an optimal distributions.*

Note, that using the Poisson equation, the exterior electric field can be retrieved. Also, in view of the fact that interior and exterior charged appear additive measures the algorithm constructing is essentially facilitate.

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ON POLE ASSIGNMENT IN BILINEAR CONTROL SYSTEMS

Consider a linear stationary control system

$$\dot{x} = A_0x + Bu, \quad y = C^*x, \quad (x, u, y) \in \mathbf{R}^n \times \mathbf{R}^m \times \mathbf{R}^k. \tag{1}$$

Let the control in system (1) be constructed as linear incomplete feedback $u = Uy$. Corresponding closed-loop system is

$$\dot{x} = (A_0 + BUC^*)x, \qquad x \in \mathbf{R}^n. \tag{2}$$

Consider a bilinear stationary control system also

$$\dot{x} = (A_0 + u_1A_1 + \ldots + u_rA_r)x, \qquad x \in \mathbf{R}^n. \tag{3}$$

System (3) is more general than system (2). We say that *the pole assignment problem in the system (3) is solvable*, if for any given polynomial $p(\lambda) = \lambda^n + \gamma_1\lambda^{n-1} + \ldots + \gamma_n$, $\gamma_i \in \mathbf{R}$ there exists a constant control $u = (u_1, \ldots, u_r) \in \mathbf{R}^r$ such that the characteristic polynomial $\chi(A_0 + u_1A_1 + \ldots + u_rA_r; \lambda)$ of the matrix $A_0 + u_1A_1 + \ldots + u_rA_r$ coincides with $p(\lambda)$. The necessary and sufficient conditions of pole assignment problem solvability have been obtained in [1, 2] for system (2) provided that the first $(p-1)$ rows of the matrix B and the last $(n-p)$ rows of the matrix C are equal to zero for some $p \in \{\overline{1,n}\}$. This result is extended to system (3) here.

Suppose that the coefficients of the system (3) satisfy the following conditions: $A_0 = \{a^0_{ij}\}^n_{i,j=1}$; $a^0_{i,i+1} \neq 0$, $i = \overline{1,n-1}$; $a^0_{ij} = 0$, $j > i+1$, $i = \overline{1,n-2}$; the first $(p-1)$ rows and the last $(n-p)$ columns of the matrices A_s, $s = \overline{1,r}$ are equal to zero for some $p \in \{\overline{1,n}\}$. Suppose $\chi(A_0; \lambda) = \lambda^n + \alpha_1\lambda^{n-1} + \ldots + \alpha_n$. Let us construct the matrix $S_1 = \{s^1_{ij}\}^n_{i,j=1}$ from the matrix A_0: $s^1_{11} := 1$; $s^1_{1j} := 0$, $j = \overline{2,n}$; $s^1_{ij} := a^0_{i-1,j}$, $i = \overline{2,n}$, $j = \overline{1,n}$. Then we construct the matrix $S_l = \{s^l_{ij}\}^n_{i,j=1}$ from the matrix $S_{l-1} = \{s^{l-1}_{ij}\}^n_{i,j=1}$ for every $l = \overline{2,n}$ in the following way: $s^l_{11} := 1$, $s^l_{1j} := s^l_{j1} := 0$, $j = \overline{2,n}$; $s^l_{ij} := s^{l-1}_{i-1,j-1}$, $i,j = \overline{2,n}$. Let $S = S_n \cdot S_{n-1} \cdot \ldots \cdot S_1$. All the matrices S_l and S are nonsingular lower triangle. Suppose $J_1 = \{g_{ij}\}^n_{i,j=1}$; $g_{i,i+1} = 1$, $i = \overline{1,n-1}$; $g_{ij} = 0$, $j \neq i+1$; $J_k := J_1^k$; $J_0 := I$. Let us construct $G := \sum\limits_{i=1}^{n} \alpha_{i-1}J^*_{i-1}$; $\alpha_0 := 1$. Let $\widetilde{A}_i := SA_iS^{-1}$, $i = \overline{0,r}$. Then $\widetilde{A}_0 = J_1 + e_n \cdot \xi$ where $e_n = \operatorname{col}(0, \ldots, 0, 1) \in \mathbf{R}^n$, $\xi = (-\alpha_n, \ldots, -\alpha_1) \in \mathbf{R}^{n*}$ and the matrices $\widetilde{A}_i$, $i = \overline{1,r}$ have the same structure as A_i, $i = \overline{1,r}$. One has $\chi(A_0 + u_1A_1 + \ldots + u_rA_r; \lambda) = \chi(\widetilde{A}_0 + u_1\widetilde{A}_1 + \ldots + u_r\widetilde{A}_r; \lambda)$. Let $\widetilde{a}^i_j \in \mathbf{R}^n$ be the j-th column of the matrix $\widetilde{A}_i$, $i = \overline{1,r}$. Let us construct the $(n \times r)$-matrices $P_1 = [\widetilde{a}^1_1, \ldots, \widetilde{a}^r_1]$, ..., $P_n = [\widetilde{a}^1_n, \ldots, \widetilde{a}^r_n]$ from the $(n \times n)$-matrices $\widetilde{A}_i = [\widetilde{a}^i_1, \widetilde{a}^i_2, \ldots, \widetilde{a}^i_n]$, $i = \overline{1,r}$. Then we construct the $(n \times r)$-matrix $Q = J_0GP_1 + J_1GP_2 + \ldots + J_{n-1}GP_n$.

Theorem 1. Suppose $\chi(A_0 + u_1A_1 + \ldots + u_rA_r; \lambda) = \lambda^n + \gamma_1\lambda^{n-1} + \ldots + \gamma_n$. Then

$$\gamma = \alpha - Qu \tag{4}$$

where $\gamma = \operatorname{col}(\gamma_1, \ldots, \gamma_n)$, $\alpha = \operatorname{col}(\alpha_1, \ldots, \alpha_n)$, $u = \operatorname{col}(u_1, \ldots, u_r)$.

Theorem 2. Suppose $\chi(A_0 + BUC^*; \lambda) = \lambda^n + \gamma_1\lambda^{n-1} + \ldots + \gamma_n$. Then

$$\gamma_i = \alpha_i - \operatorname{Tr} SBUC^*S^{-1}J_{i-1}G, \quad i = \overline{1,n}. \tag{5}$$

Theorem 3. **a)** The pole assignment problem in the system (3) is solvable if and only if the rows of the matrix Q are linearly independent. In that case the control u which reduces $\chi(A_0 + u_1A_1 + \ldots + u_rA_r; \lambda)$ to the given polynomial $p(\lambda)$ with the coefficients γ_i is found from the system (4). **b)** The pole assignment problem in the system (2) is solvable if and only if the matrices

$$C^*S^{-1}J_0GSB,\ C^*S^{-1}J_1GSB,\ \ldots,\ C^*S^{-1}J_{n-1}GSB \tag{6}$$

are linearly independent. In that case the control U which reduces $\chi(A_0 + BUC^*;\lambda)$ to the given polynomial $p(\lambda)$ with the coefficients γ_i is found from the system (5).

Corollary 1. If the rows of the matrix Q are linearly independent then the system (3) is stabilizable by the constant control $u \in \mathbf{R}^r$. If the matrices (6) are linearly independent then the system (2) is stabilizable by the constant matrix control U.

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ON PSEUDO-ANOSOV HOMEOMORPHISMS WITH SMALL DILATATION

Pseudo-Anosov (PA) homeomorphisms f of closed surfaces M_g^p (g — genius, p — the number of punctures) are considered. It is known that the dilatation λ (i.e. the stretch factor of the expansion of unstable foliation and $\log\lambda$ is the topological entropy of f) of PA-homeomorphism is bounded from below by some number $\geqslant 1$. The question arises: which is the minimum of dilatation for PA-homeomorphisms of given surface? The exact answer is known for several orientable surfaces (with $g \leqslant 2$ and $p = 0$ and $g = 0$ and $p \leqslant 6$). It is known also that the minimum of dilatation tends to 1 if either $p \to \infty$ and $g = 1$ ($g = 0$) for (non)-orientable surface or $p = 0$ and $g \to \infty$. In some papers (see for example [1]) the estimations of the asymptotic behavior of the minimal dilatation depending on either genius or the number of punctures is given. All these results are obtained by constructing of concrete examples or series of examples of PA-homeomorphisms. To do this different methods where used: train-tracks, braids and so on.

In the talk the review of known and some new (especially for the case of non-orientable surfaces) examples will be given. The method of constructing them is based on the method of combinatorial description of Markov partitions for PA-homeomorphisms elaborated by author for the purpose of classification of diffeomorphisms of surfaces with hyperbolic attractors [2].

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EXCHANGE TRANSFORMATIONS WITH FLIPS

We consider the problem of existence of uniquely ergodic exchange transformations with flips on a circle.

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НЕКОТОРЫЕ ОБРАТНЫЕ ЗАДАЧИ ДЛЯ ПАРАБОЛИЧЕСКИХ УРАВНЕНИЙ
SOME INVERSE PROBLEMS FOR PARABOLIC EQUATIONS

В работе исследуется следующая обратная задача определения правой части специального вида

Задача. Найти пару функций $u(x,t)$ и $q(x,t)$, удовлетворяющих уравнению

$$u_t - \Delta u = q(x,t)\Phi(x,t) + f(x,t), \quad (x,t) \in Q,$$

и однородным краевым условиям

$$u(x,0) = u(x,T) = 0, \quad x \in \Omega,$$
$$u|_\Gamma = \frac{\partial u}{\partial n}\Big|_\Gamma = 0.$$

Здесь $\Omega \subset \mathbb{R}^n$ — ограниченная область с гладкой границей, $Q = \Omega \times (0,T)$, $T < \infty$, $\Gamma = \partial\Omega \times (0,T)$.

Доказаны теоремы существования и единственности решения в случае, когда функция $q(x,t)$ лежит в ядре дифференциального оператора $L \equiv \frac{\partial}{\partial t} + \gamma\Delta$ ($\gamma > 0$).

В случае, когда правая часть имеет произвольный вид, задача исследовалась с помощью метода введения параметра. А именно, исследовалась следующая