SAINT PETERSBURG STATE UNIVERSITY
Sobolev Astronomical Institute

STELLAR DYNAMICS:
FROM CLASSIC TO MODERN

Proceedings of the International Conference
held in Saint Petersburg, August 21–27, 2000,
in honour of the 100th birthday of Professor
K. F. Ogorodnikov (1900–1985)

Edited by
L. P. Ossipkov and I. I. Nikiforov
Saint Petersburg State University, Russia

Saint Petersburg
2001

The volume contains the proceedings of the International Conference "Stellar Dynamics: from Classic to Modern" held in Saint Petersburg, August 21–27, 2000, in honour of the centennial of Professor Kyrill Fedorovich Ogorodnikov's birth.

Publication is supported by the State Program for Leading Scientific Schools of Russia (grant number 00-15-96775).

SCIENTIFIC ORGANIZING COMMITTEE

T. A. Agekian	(Honorary Chair)	Russia
C. Hunter	(Co-Chair)	USA
S. A. Kutuzov	(Co-Chair)	Russia
L. P. Ossipkov	(Deputy Chair)	Russia
I. I. Nikiforov	(Secretary)	Russia

V. A. Antonov (Russia), E. Athanassoula (France), J. Binney (UK), A. D. Chernin (Russia), G. Contopoulos (Greece), T. de Zeeuw (The Netherlands), H. Dejonghe (Belgium), K. C. Freeman (Australia), A. M. Fridman (Russia), O. Gerhard (Switzerland), K. Kuijken (The Netherlands), D. Merritt (USA), E. M. Nezhinskij (The Netherlands), S. N. Nuritdinov (Uzbekistan)

LOCAL ORGANIZING COMMITTEE

N. P. Pitjev (Chair), V. V. Orlov (Deputy Chair), A. V. Elkin, M. A. Mardanova, L. L. Sokolov

ISBN 5-7997-0290-5

Технические редакторы — А. В. Елькин, И. И. Никифоров

Equilibrium Figures of Gas-Dust Clouds in the Galaxy

B. P. Kondratyev

Mathematical Department, Udmurtia State University, Universitetskaya ul. 1, Izhevsk 426034, Russia

Abstract. In the tidal approximation the problem of the equilibrium form of dense globules is considered. Modeling a globula by a homogeneous triaxial ellipsoid and setting the potential in the symmetry plane of the Galaxy the equilibrium equations for it were obtained. It is shown that it is possible to neglect the tidal component of the Galaxy attraction in z-direction. It follows from these equations that in our Galaxy a globule can have a form of only a prolated spheroid (aligned to the galactic center). A meridional flattening of the globule is obtained as a function of its location in the Galaxy.

1. Introduction

According to observations, in our Galaxy there exist numerous rather dense, compact gas-dust nebulas, which Bok & Reilly (1947) called as globules. So, more than 200 such objects are revealed in radius 500 pc. The globules have the characteristic sizes from 0.2 up to 0.6 pc and masses from 20 up to 200 $M_\odot$. The physical conditions in these dense nebulas are such, that young stars can be formed there. Form of many of nebulas is smoothed, and sometimes they are round. Hence, they are in equilibrium state under influence of external and internal forces.

It seems that the reasons forcing a globule to reach the regular form were not strictly considered by anybody. There are some hints (Dibaj 1972), that light pressure from environmental hot gas plays role in globule formation, but they do not clarify the problem, and the more complete consideration of the problem about form of such nebulas with taking into account all physical factors is necessary. Concrete estimations show, that usual mechanical forces of inertia and gravitation play the basic role in formation of globules. Some elements of studying such configurations by the virial method are in a paper by Kuzmin (1964).

Here the problem of the equilibrium form of dense globules is considered in tidal approximation.

2. The Equilibrium Figures of Globules

We consider a compact globule in the equatorial plane of the Galaxy at the distance R_0 from its center. Let the globule moves around it along a circular orbit with an angular velocity $\Omega(R_0)$. We denote $\Phi(R)$ the gravitational potential of the Galaxy in the equatorial plane. It follows from balance of gravitational and centrifugal forces that

$$\Omega^2 R_0 = -\left(\frac{\partial\Phi}{\partial R}\right)_0 . \qquad (1)$$

Compactness of the globule allows to expand the potential $\Phi(R, z)$ in a power series at the point R_0 in small $R - R_0$ and z up to squares. For internal points of the cloud

$$R = \sqrt{x^2 + (y + R_0)^2} = (R_0 + y)\sqrt{1 + \frac{x^2}{(y + R_0)^2}} \approx R_0 + y + \frac{x^2}{2R_0}. \qquad (2)$$

Then, taking into account Eq. (1) we obtain

$$\Phi(R_0, z) = \Phi_0 - \Omega^2 R_0 \left(y + \frac{x^2}{2R_0}\right) + \frac{1}{2}\left(-\kappa^2 + 3\Omega^2\right) y^2 + \frac{1}{2}\left(\frac{\partial^2\Phi}{\partial z^2}\right)_0 z^2. \qquad (3)$$

Here we introduced the square of epicyclic frequency

$$\kappa^2 = -\left[\frac{3}{R_0}\left(\frac{\partial\Phi}{\partial R}\right)_0 + \left(\frac{\partial^2\Phi}{\partial R^2}\right)_0\right] . \qquad (4)$$

Further, let model the equilibrium gas-dust cloud by the homogeneous triaxial ellipsoid with semi-axes a, b, c and the internal potential

$$\varphi = \pi G\rho \left(I - A_1 x^2 - A_2 y^2 - A_3 z^2\right) . \qquad (5)$$

with well-known factors A_i.

The total reduced potential in internal point of the nebula will be as follows

$$W = \Phi_0 - \Omega^2 R_0 \left(y + \frac{x^2}{2R_0}\right) +$$

$$\left(3\Omega^2 - \kappa^2\right)\frac{y^2}{2} + \frac{\Omega^2}{2}\left[x^2 + (y + R_0)^2\right] + \tag{6}$$
$$\pi G\rho\left(I - A_1x^2 - A_2y^2 - A_3z^2\right) + \frac{1}{2}\left(\frac{\partial^2\Phi}{\partial z^2}\right)_0 z^2,$$

or, after obvious transformations and reductions,

$$W = \Phi_0 + \left(2\Omega^2 - \frac{\kappa^2}{2}\right)y^2 + \frac{\Omega^2R_0^2}{2} +$$
$$\pi G\rho\left(I - A_1x^2 - A_2y^2 - A_3z^2\right) + \frac{1}{2}\left(\frac{\partial^2\Phi}{\partial z^2}\right)_0 z^2. \tag{7}$$

According to the theory of equilibrium figures, a surface of a stationary globule should coincide with one of level surfaces. This condition yields

$$W = -\lambda\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}\right) = -\pi G\rho A_1x^2 +$$
$$\left(2\Omega^2 - \frac{\kappa^2}{2} - \pi G\rho A_2\right)y^2 + \left[\frac{1}{2}\left(\frac{\partial^2\Phi}{\partial z^2}\right)_0 - \pi G\rho A_3\right]z^2, \tag{8}$$

where λ is the constant factor. The factors at x^2, y^2 and z^2 must vanish that leads to the following system of two equations:

$$a^2A_1 = b^2\left(A_2 - \frac{2\Omega^2}{\pi G\rho} + \frac{\kappa^2}{2\pi G\rho}\right) = c^2\left[A_3 - \frac{1}{2\pi G\rho}\left(\frac{\partial^2\Phi}{\partial z^2}\right)_0\right]. \tag{9}$$

It should be pointed out here that for real average globule density $\rho \approx 4\times10^{-21}$ g m^{-3} and Oort's estimate for $\left(\partial^2\Phi/\partial z^2\right)_0 \approx -9.21\times10^{-30}$ s^{-2} (Lindblad 1959) the term $\left(\partial^2\Phi/\partial z^2\right)_0/(2\pi G\rho) \approx 5.5\times10^{-3}$ is considerably less then the value A_3 ($\geq 2/3$). Hence, the last term in the right hand side of Eq. (9) can be rejected, and with good approximation we receive from Eq. (9)

$$a^2A_1 = b^2\left(A_2 - \frac{2\Omega^2}{\pi G\rho} + \frac{\kappa^2}{2\pi G\rho}\right) = c^2A_3. \tag{10}$$

First of all, it follows from Eq. (10) that $a = c$, i.e., the equilibrium figure of globule is a spheroid with the symmetry axis Oy. Further, we find from Eq. (10) that

$$\frac{d^2\Phi}{dR^2} - \frac{1}{R}\frac{d\Phi}{dR} = 2K, \tag{11}$$

where index "0" is omitted. Here

$$K = \pi G \rho \left(A_2 - \frac{a^2}{b^2} A_1 \right), \tag{12}$$

and the gravitational potential is taken at the equatorial plane of the Galaxy.

The Eq. (12) can be written down also as the following

$$K = -R\Omega \frac{d\Omega}{dR}, \tag{13}$$

or

$$K = \frac{v_{rot}}{R} \left[\frac{v_{rot}}{R} - \frac{dv_{rot}}{dR} \right], \tag{14}$$

where v_{rot} is the rotation velocity at distance R. If we express v_{rot} in $\mathrm{km\,s^{-1}}$, and R in kpc, it follows from Eq. (14) that for average density of globule $\rho \approx 4 \times 10^{-21}$ $\mathrm{g\,cm^{-3}}$

$$A_2 - \frac{a^2}{b^2} A_1 = 1.25 \times 10^{-6} \frac{v_{rot}}{R} \left[\frac{v_{rot}}{R} - \frac{dv_{rot}}{dR} \right]. \tag{15}$$

The left part in Eq. (15) depends only on flattening of spheroidal globule (Figure 1).

The right hand side in Eq. (15) is a function of R. For a wide spectrum of rotation laws of real galaxies (Rubin 1979, Marochnik & Suchkov 1984) it will be a non-negative function of R. The latter means, that globules in our Galaxy can has a form only of prolate spheroids and never oblate. We have carried out calculations of the right hand side in Eq. (15) for the real rotation law of our Galaxy (Figure 2).

Thus, the flattening of globule depends on distance R of its distance from the galactic center. The results of calculations according to Eq. (15) are shown in Figure 3.

We see, that on distance $R \approx 0.8$ kpc the prolated spheroid has the relation $a/b \approx 0.6$ (the point A). Closer that distance the globules in the Galaxy should not exist (however, this value 0.8 kpc is a model result). With increasing the distance from the center to the periphery of Galaxy flattening of globules can vary in two ways: either ones are fast rounded, or quickly get the form of the rather prolated spokes. Hence, steady globules in our Galaxy located further than 2–3 kpc and more from its center should have rather prolated or almost spherical form. The first seems to be unstable and inclined to destruction. Apparently, the globules at the Solar neighbourhood should be very close to sphere.

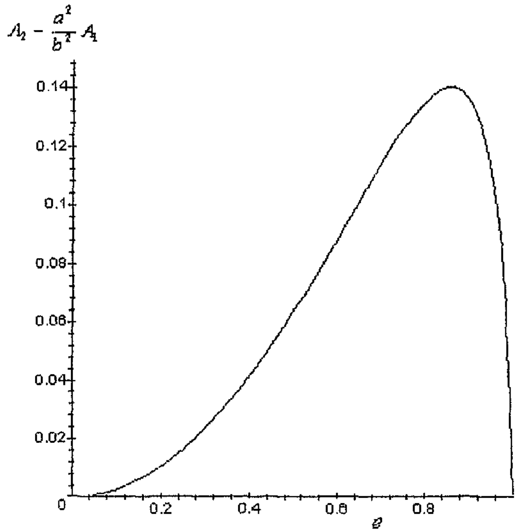

Figure 1. The dependence of the left hand side in Eq. (15) on the eccentricity e.

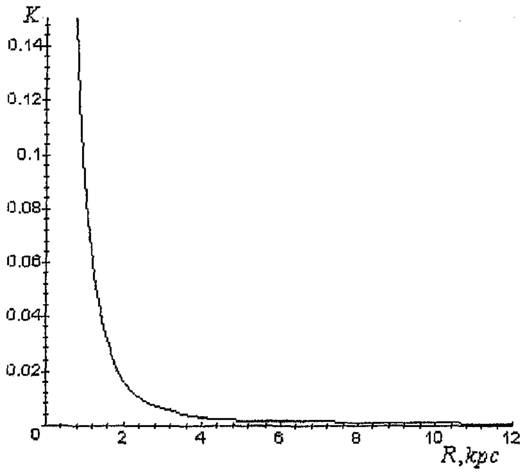

Figure 2. The dependence of the right hand side in Eq. (15) on the distance R.

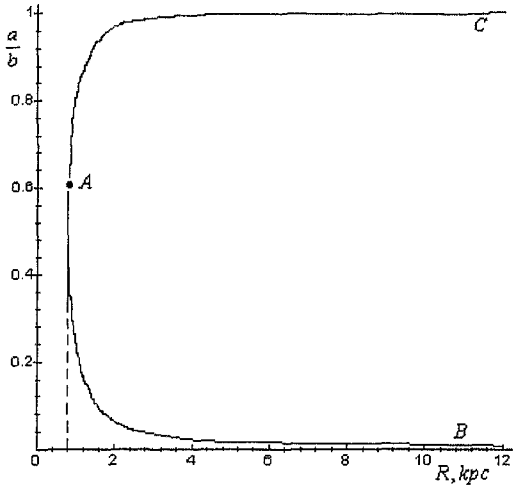

Figure 3. The relation of semiaxes as a function of R. The curve AC contains figures close to a sphere; the curve AB shows very prolate spheroidal forms of globules.

Acknowledgments. I am grateful to L. P. Ossipkov for his comments on this paper.

References

Bok, B. J., & Reilly, E. 1947, ApJ, 105, 255

Dibaj, E. A. 1972, AZh, 1134

Kuzmin, G. G. 1964, Publ.Tartusk.astron.observ., 34, 9

Lindblad, B. 1959, in Handbuch der Physik. Band LII. Astrophysik IV: Sternsysteme, S. Flügge, Berlin: Springer, 21

Marochnik, L. S., & Suchkov, A. A. 1984, The Galaxy, Moscow: Nauka

Rubin, V. C. 1979, in The Large-Scale Characteristics of the Galaxy, W. B. Burton, Dordrecht: Reidel, 211