INHERITANCE OF SOME PROPERTIES BY GREEN OPERATORS OF DIFFERENTIAL EQUATIONS UNDER LINEAR PERTURBATIONS

Galimzyan G. Islamov, Udmurt State University (Russia)

Suppose that T is a compactum in the coordinate space R^m, $C = C(T)$, $B = L(T)$ is the Banach space of scalar functions integrable on T, D is a subspace of B isomorphic to the direct product $B \times R^n$, and $Q : D \to C$ is a weakly compact linear injective mapping.

Assume that for any finite system of distinct points $t_i \in T, i = 1, \dots, \nu$ the interpolation problem $(Qx)(t_i) = \beta_i, i = 1, \dots, \nu$ is solvable in D for any $\beta_i \in R, i = 1, \dots, \nu$.

Suppose, further, that $L : D \to B$ is a bounded linear operator, and $l_i, i = 1, \dots, n$ are linearly independent functionals in the dual space D^*.

Assume that the bounded value problem $Lx = f, l_i x = 0, i = 1, \dots, n$ is uniquely solvable in D for any $f \in B$, and the operator $W : B \to D$ assigning to f the solution $x = Wf$ of this problem (the Green operator) has the property that for some linear homeomorphism H of the space B the product $QWH : B \to C$ is a positive operator with respect to the cones of nonnegative functions in the spaces B and C, i.e. $QWH > 0$.

Theorem. *Suppose that $V : C \to B$ is a positive linear operator, there is a function $u \in D$ such that $Qu \geq 0$ and $l_i u = 0, i = 1, \dots, n$ and the residual $\psi = H^{-1}Lu - VQu$ is positive almost everywhere in T. Then the perturbed boundary value problem $Lx = HVQx + f, l_i u = 0, i = 1, \dots, n$ is also uniquely solvable in D for any $f \in B$, and its Green operator G inherits the property of W, namely, $QGH > 0$.*

Completeness of root vectors and degree of approximation by finite-dimensional operators are next properties which can be taken into considerations (*see G.G. Islamov, Some applications of the theory of abstract functional-differential equations I;II //Differ. equations, 25, No. 11, 1309-1317 (1989); 26, No. 2, 167-173 (1990); Estimation of the minimal rank of finite-dimensional perturbations of Green's operators //Differ. equations, 25, No. 9, 1046-1052 (1989)).*