ACADEMY OF SCIENCES OF THE USSR

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Space Science Preprint

No. 143

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TWO-COMPONENT MODEL OF A SPHERICAL STELLAR SYSTEM

Moscow, 1981

ABSTRACT

As a generalization of the widely used King's model for a spherical stellar system containing stars of identical masses, a two-component model of an equilibrium stellar system consisting of stars of two different masses is built proceeding from a proper generalization of the distribution function. In such a system, the principle of thermal equilibrium,generally speaking, does not hold, i.e. equipartition of mean kinetic energy between light and heavy stars is absent

The radial distribution of partial density and velocity dispersion for each of the components and of the total density and velocity dispersion is obtained. The profiles of both the total density and velocity dispersion differ considerably from those given by the one-component King's model, especially in the centre of the system. In particular, the velocity dispersion of stars may have there a non-monotonous behaviour. The possibility to apply the many-component model for the explanation of the recently deiscovered luminosity cups in the central regions of some elliptical galaxies is briefly discussed.

1. Introduction.

In investigations of stationary stellar systems an important role is played by the choice (finding) of the phase distribution function. If the latter is known, it is easy to construct a dynamical model of a stellar system.

A successful step in this direction is the self-consistent King's model (1966) based on an analytical approximation (Michie, 1963; King, 1965) to the stationary solution of the Fokker-Planck equation (Spitzer and Härm, 1958); this approximation corresponds to the Gaussian velocity distribution function (subtracting a constant which nullifies the distribution function for stars with energies $E \geqslant 0$):

$$f(E) = C \cdot \left[\exp(-2j^2 E) - 1 \right] \cdot \qquad \text{(I)}$$

Here $E = v^2/2 - \Phi(r)$ is the integral of energy for a star (per unit mass); j is a parameter which determines the velocity dispersion of stars (in the case of the Maxwell distribution, the latter is equal to $(3/2)j^{-2}$); $\Phi(r)$ is a shifted gravitational potential (i.e. a potential so determined that it vanishes at the boundary of the stellar system). Expression (I) contains an additional assumption that all the stars have one and the same mass.

The model based on the distribution function (I) is free from some serious defects inherent in the ordinary isothermic model, and with a proper choice of the concentration parameter $\lg(r_t/r_c)$ (where r_c is the core radius , and r_t is the tidal radius of the stellar system) this model describes satisfactorily the observed surface brightness' profile in many globular clusters and in some elliptical galaxies.

In a more detailed study of real stellar systems, the situation occurs more and more often when the distribution function (I), describing well the profile of brightness in external parts of a system, reproduces it badly in the central region. As natural attempts to remove such a discordance there appeared extensions of the King's model to the case of stars of different masses (Da Costa and Feeman, 1976; Illingworth and King, 1977).

It should be emphasized that introducing into the consideration of stars of different masses is not at all a trivial procedure. A difference in the star masses leads to a number of new effects. Long ago Spitzer (1969) noted that stars of two masses m_I and m_2 which constitute a spherical stellar system are not in equipartition with each other in a rather wide interval of parameters of the system. Saslaw and De Young (1971) and Vishniac (1978) revealed that the same effect takes place also at a continuous mass distribution of stars. The difference in the masses of the stars constituting a stellar system leads to a noticeable shortening of the duration of its dynamical evolution (Spitzer and Hart, 1971; Lightman, 1977).

While the King model with equal-mass stars (a "one-component model") has been studied rather thoroughly, its many-component generalizations, including even the simpliest case of two masses (a "two-component model") have been investigated very little. Meanhwile, one-and many-component models may differ strongly in their properties and, in particular, in the profiles of both the spatial star density and star velocity dispersion. Some qualitative features of the behaviour of the velocity dispersion in a two-component model were considered by Binney (1980), who did not deal with the distribution functions but supposed that de Vaucouleurs' law $r^{1/4}$ remains valid also within the internal regions of stellar systems. It is more consistent, however, to proceed from the distribution function. On the basis of such an approach Da Costa and Freeman (1976) using the many-component King's model, could achieve an agreement between their theory and observations for the globular cluster M3. This model was based on the phase functions for each component taken in the form

$$f_i(E) = A_i\left[\exp(-E/\sigma_i^2) - 1\right] , \qquad (2)$$

where σ_i is the velocity dispersion of the i-th component of the stellar population. Besides, they supposed that there exists equipartition of mean kinetic energy among stars of different masses, i.e.

$$m_1\sigma_1^2 = m_2\sigma_2^2 = \dots \qquad (3)$$

However, such an approach is, strictly speaking, inconsistent. Indeed, the condition (3) is rigorously fulfilled in a many-component isothermic system only, i.e. when partial distribution functions have the form $f_i(E) \sim \exp(-E/const)$, which differs from Equation (2). On the other hand, if the distribution function is taken in the form (2) and used for the calculation of the velocity dispersion, then, as is shown in the Appendix, the condition of hydrostatic equilibrium is not fulfilled for each class of stars, which is inadmissible *

In the present work, we have chosen another approach. We take the phase distribution functions in such a manner that for each stellar component (and, therefore, for the system as a whole) the requirement of hydrostatic equilibrium is being held. An equilibrium system when constructed for the most simple case of the two-component model shows that, despite the presence of a hydrostatic equilibrium, the thermal equilibrium (i.e. equipartition of mean kinetic energy between stars of different masses) may be absent. In this case, the equalities (3) are violated even at the centre of the system, and when approaching its boundary this violation increases. The less is the (dimensionless) potential in the centre of the system, the more is the deviation from the thermal equilibrium. [At the same time,

* True, under conditions of a typical globular cluster, the value of the dimensionless potential at the centre of the system is such that deviations from hydrostatic

(cont. on p.6)

under the transition to an isothermic many-component system (which is achieved by tending of the central potential to infinity) there occurs an equipartition between the components. The reason is that with the increase of the central potential the influence of the boundary of a star cluster on the dynamics of the latter decreases. Clearly, in the limiting case of the isothermic sphere this influence vanishes completely]. As is shown in this paper, a deviation from the thermal equilibrium is related tightly to the character of distribution of light and heavy stars as functions of the distance from the centre of the system and, for instance, it determines the sign of the derivative of the star velocity dispersion in the vicinity of the centre.

The content of the paper is as follows. The fundamental formulae for the two-component model are obtained in Section 2. These formulae are used for numerical calculations of the star density and velocity dispersion profiles in Section 3. In Section 4, we discuss the problem of the absence of energy equipartition between the stars as well as the connection of this effect with the character of star density and velocity dispersion distributions. In Section 5, main conclusions of the paper are summarized and possible applications are outlined.

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equilibrium are rather small, and the model may be considered to be close to isothermical one. In this case, the equalities (3) are approximately valid.

2. The two-component model.

Generalizing the one-component King's model with the distribution function given by Equation (1), we will write the phase functions for two star components with the masses m_1 and $m_2 \geqslant m_1$ in the form

$$f_1(E) = A_1 \cdot \left[exp\left(-\frac{3Em_1}{\beta}\right) - 1 \right] ,$$
$$f_2(E) = A_2 \left[exp\left(-\frac{3Em_2}{\beta}\right) - 1 \right] , \quad (4)$$

where E is, as previously, the integral of energy (per unit mass).

As in the model associated with Equation (1) those stars that have $E \geqslant 0$ are removed from the system by an external tidal field. The tidal radius r_t is identical for both groups of stars as it does not depend on the star mass.

As distinct from the distribution functions given by Equation (2), in Equations (4) the energy in the exponent is normalized not to the velocity dispersion of the i-th component, but to the factor β/m_i. This change in the form of phase functions, insignificant at the first sight, has far-reaching physical consequences, since unlike Equation (2) it provides the possibility of constructing a hydrostatically equilibrium model (see below Equation (14)).

The profiles of star density and velocity dispersion for each component can be found using the usual expressions:

$$\rho_i = m_i \int_0^{\sqrt{2\Phi}} f_i(E)\, d^3v \; ;$$

$$\sigma_i^2 = \left(\int_0^{\sqrt{2\Phi}} v^2 f_i(E)\, d^3v \right) \Big/ \left(\int_0^{\sqrt{2\Phi}} f_i(E)\, d^3v \right) . \tag{5}$$

Introducing the dimensionless potential $U(r)$ and the heavy to light star's mass ratio μ

$$U(r) \equiv \frac{3m_1}{\beta} \Phi(r) \; ; \quad \mu \equiv m_2/m_1 \; , \tag{6}$$

we obtain

$$\rho_1 = \rho_{01} \cdot I(U)/I(U_0) \, ,$$
$$\rho_2 = \rho_{02} \cdot I(\mu U)/I(\mu U_0) \, , \tag{7}$$

where the function $I(U)$ is defined by Equation (9) (see below); U_0 is the central value of the potential, and

$$\rho_{01} = 2\pi A_1 m_1 \left(\frac{2\beta}{3m_1}\right)^{3/2} I(U_0) \, ,$$
$$\rho_{02} = 2\pi A_2 m_2 \left(\frac{2\beta}{3m_2}\right)^{3/2} I(\mu U_0) \, .$$

Equation (5) yields also

$$\sigma_1^2 = \frac{\beta}{m_1} \left(1 - \frac{4}{15} \frac{U^{5/2}}{I(U)} \right) ,$$
$$\sigma_2^2 = \frac{\beta}{m_2} \left(1 - \frac{4}{15} \frac{(\mu U)^{5/2}}{I(\mu U)} \right) , \tag{8}$$

where the difference, for example, between σ_1^2 and β/m_1 is written in an explicit form. This difference increases with decreasing U. On the contrary, at $U \to \infty$ it becomes vanishingly small. In Equations (7)-(8) we define:

$$I(U) \equiv \frac{\sqrt{\pi}}{2} \, erf(U^{1/2}) \exp(U) - \frac{2}{3} U^{3/2} - U^{1/2} \, , \tag{9}$$

where

$$\mathrm{erf}(U^{1/2}) = \frac{2}{\sqrt{\pi}}\int_0^{U^{1/2}} e^{-t^2}dt$$

is the error integral. For further calculations the following relation is useful

$$\frac{d\,I(U)}{dU} = I(U) + \frac{2}{3}U^{3/2}.$$

Introducing the ratio of the central densities of the heavy and light components

$$\lambda = \rho_{02}/\rho_{01} \quad , \tag{10}$$

we write the total density in the form

$$\rho_t = \rho_{01}\left[\frac{I(U)}{I(U_0)} + \lambda\frac{I(\mu U)}{I(\mu U_0)}\right] . \tag{11}$$

As to the total velocity dispersion σ_t^2 , we have analogously to the Dalton's law for a mixture of gases:

$$\rho_t\sigma_t^2 = \rho_1\sigma_1^2 + \rho_2\sigma_2^2 \quad , \tag{12}$$

whence

$$\sigma_t^2 = \frac{\beta}{m_1}\left\{\frac{\frac{1}{I(U_0)}\left[I(U) - \frac{4}{15}U^{5/2}\right] + \frac{\lambda}{\mu I(\mu U_0)}\left[I(\mu U) - \frac{4}{15}(\mu U)^{5/2}\right]}{\frac{I(U)}{I(U_0)} + \lambda\frac{I(\mu U)}{I(\mu U_0)}}\right\}. \tag{13}$$

It is easily verified that each component separately, as well as the model as a whole, is in the state of hydrostatic equilibrium, i.e. the solutions found satisfy the Equation

$$\frac{d}{dr}(\rho_i b_i^2) = \rho_i(r)\frac{\beta}{m_1}\frac{dU(r)}{dr} \quad . \tag{14}$$

By introducing the dimensionless length

$$x = \sqrt{\frac{12\pi G m_1 \rho_{01}}{\beta}} \cdot r \quad ,$$

the Poisson equation takes the form

$$\frac{d^2U}{dx^2} + \frac{2}{x}\frac{dU}{dx} = -\frac{I(U)}{I(U_0)} - \lambda\frac{I(\mu U)}{I(\mu U_0)} \tag{15}$$

The boundary conditions for U are evident:

$$U = U_0 \ , \quad \frac{dU}{dx} = 0 \quad \text{at } x = 0 \ . \tag{16}$$

In the limiting case $\lambda = 0$ (and $\mu = I$), as is readily seen, we go back to the one-component King's model where $2j^2 = \frac{3m_1}{\beta}$.

III. Numerical results.

Equation (15) with the boundary conditions (16) has been integrated numerically by the Runge-Kutt's method. The trial calculations show that a variation of the value of the parameter $\lambda = \rho_{02}/\rho_{01}$ does not change qualitatively the character of the curves for density and velocity dispersion, and in the following computations we have restricted ourselves to two values of λ equal to zero (which corresponds to the one-component model) and $\lambda = 3$. As far as $\mu = m_2/m_1$ is concerned, this parameter was taken equal to 1, 3/2, 3 and 4. To demonstrate the basic properties of the solutions corresponding to different sets U_0, μ and λ, we shall restrict ourselves to the values of the central potential U_0 equal to 5 and 2.

a) The density run

At high values of the central potential U_0 the properties of the two-component model are determined by the effect of an increased concentration towards the centre of heavy particles, as compared with light ones. This is readily seen from Figure I corresponding to the case $U_0 = 5$. Note that at large μ the degree of concentration of the heavy component increases. In the limiting case $U \to \infty$ we are dealing already with the isothermic sphere, wherein the concentration of heavy stars, as a function of the parameter μ, is pronounced most strongly.

The consequence of the effect described is that at large distances from the centre the profile of the total density is well approximated by the partial density of light stars (see Figure I). This is particularly well seen for large μ. Such a segretation of light and heavy stars ar large U_0 demonstrates that the deviation from the energy equipartition between them is comparatively small.

It is of interest that in the two-component system the distribution of both light and heavy stars differs from the density distribution in the one-component model. Therefore the distribution of the <u>total</u> density at large distance in the case of two components differs from that in the one-component model (cf. Figures I and 2). From the physical point of view this effect is easily explained: gravitational attraction produced by the second (heavy) component makes the stars of the first component to concentrate more toward the centre. A similar effect was noticed earlier by Taff et al. (1975) for two-component isothermic sphere.

At small U_0 the picture of the relative density distribution of both components is already different (see Figure 2). The main difference is that at small U_0 the effect of the concentration of heavy stars towards the centre is not very noticeable. Both the components make a comparable contribution to the total density, as a result of which at large distances from the centre its profile is not any more approximated by the distribution of light stars.

At small U_0 the influence of μ upon the density distribution is much less than at large U_0.

The above-mentioned specific features of density distribution make us believe that at small U_0 in the two-component model there is a strong deviation from equipartition for stars of different masses. Specifically, the velocity dispersion of heavy stars, σ_2, at small U_0 has a higher value than it follows from the formula $\sigma_2 = \sigma_1\sqrt{m_1/m_2}$. Moreover, σ_2 may even come very close to σ_1. All these properties of σ_2 are corroborated below (see Section 4).

b) The total velocity dispersion

There exist two essentially different cases:

(i) σ_t monotonously decreases as the distance from the centre of the model increases.

(ii) σ_t is not monotonous function of x, it increases at first, reaches some maximum and then decreases up to zero towards the boundary of the system.

Both these cases hold at any value of U_0 (see Figures 3 and 4). From Figure 3 corresponding to $U_0 = 5$ it is seen that at $\mu = 3/2$ the case (i) is realized, and at $\mu = 3$ we have the case (ii). A similar picture can be seen in Figure 4 corresponding to $U_0 = 2$: at $\mu = 3/2$ and $\mu = 3$ the case (i) is realized, but at $\mu = 5$ the velocity dispersion is already a non-monotonous function of the coordinate x.

If the central potential U_0 is sufficiently large, then at large distances from the centre the velocity dispersion is well approximated by the velocity dispersion of light stars. From Figure 3 it is seen that this effect is more pronounced at large μ.

It is of importance to note that at each U_0 there exists a certain critical value, μ_{cr}, such as if $\mu < \mu_{cr}$ the total velocity dispersion corresponds to the case (i) while at $\mu > \mu_{cr}$ it corresponds to the case (ii). The dependence of μ_{cr} on U_0 is shown in Figure 5. The less is U_0, the larger is μ_{cr}: at $U_0 = 5$ the value of $\mu_{cr} = 1.60$ while at $U_0 = 2$, μ_{cr} already reaches 3.43.

The reason for a different behaviour of the total velocity dispersion is, as is shown in Section 4, in the violation of the equipartition principle. Already from general considerations it is evident that the case (i) is possible only if the velocity dispersion of heavy stars exceeds the value determined for it by the equipartition law. Indeed, in a two component isothermic sphere, when there exists strict thermal equilibrium, i.e. $\sigma_2/\sigma_1=\sqrt{m_1/m_2}<1$, we have $d\sigma_t/dx>0$. True, in a two-component isothermic sphere both σ_1 and σ_2 are constant so that σ_t will increase not only in the centre but at any distance from it, since far from the centre there are mostly light stars with the velocity dispersion larger than that of heavy stars. In the two-component non-isothermic model under consideration the velocity dispersion always decreases at large distances from the centre, irrespective of its behaviour in the central region, and vanishes at the boundary of the system.

IV. The absence of equipartition.

Because of gravitational encounters of stars of different masses, there exists in a stellar system (as in a gas mixture where molecules of different masses collide) a tendency to equipartition. However, while in the gas mixture, after a proper relaxation, both mechanical and thermal equilibrium are established, the situation in the stellar system is different. One of the basic properties of the two-component system considered is the absence of equipartition of mean kinetic energy in stars of different masses. From Equations (8) we obtain

$$\frac{m_2\sigma_2^2}{m_1\sigma_1^2} = \left[1-\frac{4}{15}\frac{(\mu U)^{5/2}}{I(\mu U)}\right] \Big/ \left[1-\frac{4}{15}\frac{U^{5/2}}{I(U)}\right] . \qquad (17)$$

Since $I(\mu U) > I(U)$ at $\mu > 1$ (which follows from Equation (9)), the right-hand side of Equation (17) always exceeds unity. Hence, the ratio of the mean energies of translational motion is a function of U and μ only. At any U_o in the centre of the model there is a deviation from the thermal equilibrium (see Figure 6), and this deviation increases as U_o and μ decrease. This can be easily seen in the following way. In a two-component isothermic sphere (a transition to which from the King's model is realized at $U_o \rightarrow \infty$), the exact equality $m_2\sigma_2^2/m_1\sigma_1^2 = 1$ holds. But, as is seen from Figure 6, the ratio of energies just tends to unity when $U_o \rightarrow \infty$ .From Figure 6 it is seen that the influence of μ on $m_2\sigma_2^2/m_1\sigma_1^2$ is essential as long as $1 \leq \mu \lesssim 3 \div 4$, and at larger μ a variation of this parameter only slightly affects the value of the ratio of kinetic energies for stars of different masses unless U_o is very close to I. At the same time, at any μ there is a strong dependence between U_o and the ratio $m_2\sigma_2^2/m_1\sigma_1^2$ (see Figure 6) as long as U_o is not too large.

The violation of equipartition is especially large at small U_o and μ . In the limiting case when $U \rightarrow 0$ we have from Equation (17) that $m_2\sigma_2^2/m_1\sigma_1^2 \rightarrow \mu$ (see Figure 6), i.e. $\sigma_2 \rightarrow \sigma_1$, what corresponds to the maximum possible deviation from equipartition.

The deviation from equipartition is not equally sensitive to parameters U_0 and μ, and this difference is easy to see in Figure 7. When U_0 is small (for instance, at $U_0 = 1 \div 2$), the deviation from equpartition becomes noticeable (it exceeds, say, 25%) already when μ exceeds unity only by 10-20%.

If energy equipartition is absent at the centre of the model, the deviation on the periphery of the system only increases (Figure 8).

The absence of thermal equilibrium can be explained as follows. The very existence of equipartition suggests that the distribution function of each component is a Maxwellian one: $f(E) \sim \exp(-E/\mathrm{const})$. But in our case the exponent in the distribution function for each component contains not one but two terms (see Equation (4)). The less U_0, the more (at a given v) the energy E and, therefore, the less (closer to unity) the term $\exp(-3m_1 E/\beta)$. The same effect on the value of this term has a decrease of the parameter μ. Thus, the distribution functions given by Equation (4) can differ considerably, at a certain combination of the parameters, from the Maxwell functions, which results in a deviation from the thermal equilibrium between the components.

Hence in the two-component model there may exist appreciable deviations from thermal equilibrium. The influence of this effect upon density distribution at different values of U_0 has already been established above. Now let us consider the influence of this deviation on the velocity dispersion. According to Equation (12), the total velocity dispersion can be represented as the product of two terms:

$$\sigma_t^2(x) = \sigma_1^2(x) \cdot \frac{\rho_1(x) + \alpha(x)\cdot\rho_2(x)}{\rho_1(x) + \rho_2(x)} , \qquad (18)$$

where $\alpha(x) = \sigma_2^2(x)/\sigma_1^2(x)$. The first of the factors, $\sigma_1^2(x)$, decreases monotonously with x at any values of U_0, μ and λ. The value of $\sigma_1^2(x)$, at given U_0 and λ, decreases the slower the greater is μ (Figure 3). As for the second factor in Equation (18), it always increases with the increase of x, since $\alpha(x)$ increases (Figure 8). As it is seen, an increase of the second factor is stronger at large μ. As a result, the total velocity dispersion radically changes its properties with the increase of μ : at small μ (i.e. at $\mu < \mu_{cr}$) it monotonously decreases with distance from the centre, and at $\mu > \mu_{cr}$ it has some maximum and behaves non-monotonously.

5. Conclusions and possible applications

The two-component model of a spherical stellar system constructed in this paper is, of course, only the most simple approaching the realistic case of a continuous mass distribution of stars. Nevertheless, it reveals the important effect discovered by Spitzer (1969), who has shown that when the total mass of heavy stars exceeds some critical value, energy equipartition between light and heavy stars becomes impossible.

Spitzer assumed σ_1 and σ_2 to be constant in space (isothermic approximation) [*]. Hence, his analysis refers, in fact, to the two-component isothermic sphere; he looked for the condition under which equilibrium of such a sphere becomes already impossible. This condition is connected, in essence, with Antonov's criterion (Antonov, 1962) for the break-up of equilibrium of an isothermic sphere (see Lightman and Shapiro, 1978). Our model, as distinct from the Spitzer's one, is generally not isothermic, and the violation of equipartition in it, including a very substantial one, is possible even when a hydrostatic equilibrium still holds. This is achieved, generally speaking, at the expence of the break-up of the thermal equilibrium. In the hydrostatically equilibrium model presented above, the deviation from the thermal equilibrium increases (at a fixed μ) as the dimensionless potential U_0 decreases.

As is seen from what has been said above, an essential feature of two-component systems is the presence in the centre of the system of some density spike. It is most pronounced in the case of equipartition but remains essential also when equipartition is violated (see Figures 1 and 2). It is of

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[*] Though Spitzer estimated how the numerical coefficient in his criterion changes also in the case of politropic density distribution, he did not consider how this will affect the profiles of σ_1 and σ_2 (see Section IV).

interest to compare this corollary of the model with an important specific feature of many massive elliptical galaxies, which, has recently been discovered by means of light detectors of a very high resolution: the central regions of these galaxies differ considerably in their structure from the predictions of the standard King's model and exhibit peculiar peaks of luminosity cusps (Schweizer, 1979). Such a spike at the centre of a giant elliptical galaxy M87 was supposed to be explained by the presence of a supermassive black hole (Young et al., 1978; Sargent, 1978; de Vaucouleurs and Nieto, 1970). However, such an interpretation encounters serious difficulties (Gurzadyan and Ozernoy, 1980). As it was shown in that paper, the peak of brightness in M 87 can be associated with the existence of a dense stellar kernel, which can be quantitatively described, already in the framework of an isotropic spherical system, as a proper generalization of the King's model.

The two-component model of a spherical stellar system considered above also offers a principled possibility to interpret the luminosity cusp at the centre of an ellyptical galaxy as a purely stellar component. As distinct from the two-component isothermical model where $\frac{d\sigma_t}{dx} > 0$, at the centre of the system both cases are possible in our model: $\frac{d\sigma_t}{dx} \gtrless 0$ (see Section IV). An increase in the star velocity dispersion from periphery to the centre, obtained above as a consequence of our model (see Section III), is in a qualitative agreement with the observed behaviour of the velocity dispersion towards the centre of M 87 and NGC 3379 (Sargent et al., 1978). At the same time our model predicts a non-monotonous behaviour of the velocity dispersion near the centre (cf. Binney, 1980), which can be an observational test of the model when observation with a larger resolution become possible.

In Section IV, we have considered in detail the results of the absence of equipartition in energy for an equilibrium stellar system. But in the framework of equilibrium models one cannot answer the question what is responsible for the break-up of equipartition. To answer this question one should analyse initial stages of the stellar system's evolution. In particular, already the mechanism of violent relaxation (Lynden-Bell, 1967) indicates that early stages of stellar system contraction are accompanied by the tendency for establishing equality in velocity dispersions of light and heavy stars. This may evidently be one of the reasons for the absence of energy equipartition studied in detail in the present paper. The numerical experiments on the evolution of large N- body gravitational systems (e.g. Aarseth, 1974) confirm that the energy equipartition among particles of different masses does not have time to establish, and as a tendency to such an equipartition there forms a rapidly contracting core of heavy particles, which tends to achieve the state of infinite density for only several initial relaxation times. Thus, the absence of equipartition can become not only the consequence of the stellar system evolution but also one of the factors affecting its further behaviour.

Appendix

Consider a two-component model with star masses m_1 and m_2 and the phase functions

$$f_1(E) = A_1\left[\exp(-3E/\sigma_{01}^2) - 1\right],$$
$$f_2(E) = A_2\left[\exp(-3E/\sigma_{02}^2) - 1\right], \tag{A.I}$$

where σ_{01} and σ_{02} are velocity dispersions of the components at the centre of the system. To take the phase functions in the form (A.I) is the same as to assume the presence in the centre of a thermal equilibrium:

$$m_2/m_1 = \sigma_{01}^2/\sigma_{02}^2 = \mu \; . \tag{A.2}$$

With the help of Equation (A.I) we will find the density and velocity dispersion profiles using the usual expressions

$$\rho_i = m_i \int_0^{\sqrt{2\Phi}} f_i \, d^3v \; ; \; \sigma_i^2 = \int_0^{\sqrt{2\Phi}} v^2 f_i \, d^3v \tag{A.3}$$

The calculations yield

$$\rho_1 = \rho_{01}\frac{I(U)}{I(U_0)} \; , \; \rho_2 = \rho_{02}\frac{I(\mu U)}{I(\mu U_0)} \; ; \tag{A.4}$$

$$\sigma_1^2 = \sigma_{01}^2\left[I(U) - \frac{4}{15}U^{5/2}\right] \Big/ \left[I(U_0) - \frac{4}{15}U_0^{5/2}\right],$$

$$\sigma_2^2 = \sigma_{02}^2\left[I(\mu U) - \frac{4}{15}(\mu U)^{5/2}\right] \Big/ \left[I(\mu U_0) - \frac{4}{15}(\mu U_0)^{5/2}\right], \tag{A.5}$$

where

$$\rho_{0i} = 2\pi A_i m_i \left(\frac{2}{3}\sigma_{0i}^2\right)^{3/2} \quad , (i=1,2) \;;$$
$$\sigma_{01}^2 = 3\pi A_1 \left(\frac{2}{3}\sigma_{01}^2\right)^{5/2} \left[I(U_0) - \frac{4}{15}U_0^{5/2}\right];$$
$$\sigma_{02}^2 = 3\pi A_2 \left(\frac{2}{3}\frac{\sigma_{01}^2}{\mu}\right)^{5/2} \left[I(\mu U_0) - \frac{4}{15}(\mu U)^{5/2}\right]$$

with U and $I(U)$ given by Equations (6) and (9) of the main text of the paper.

From Equations (A.4) and (A.5) it is readily seen that the equation of hydrostatic equilibrium for each component

$$\frac{d}{dr}\left(\rho_i \sigma_i^2\right) = 3\rho_i \frac{d\Phi}{dr} \qquad (A.6)$$

does not hold. For the system as a whole there is no hydrostatic equilibrium either. Thus, the initial choice of the distribution functions in the form given by Equation (A.I) is incompatible with hydrostatic equilibrium of each component.

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Figure captions

Fig.1. Star density run as a function of a dimensionless distance from the centre of the system (U_0 = 5; λ = 3, μ = 3/2 and 3). Heavy lines denote the total star density, dotted lines stand for partial densities of the components ρ_1 and ρ_2 , and dashed line shows the density distribution in the one-component King's model.

Fig.2. The same as in Figure I at U_0 = 2

Fig.3. Star velocity dispersion as a function of dimensionless distance from the centre of the system (U_0 = 5; λ = 3; μ = 3/2 and 3). Heavy lines show the total velocity dispersion σ_t^2 (see Equation (18)), dotted lines show profiles for partial velocity dispersions σ_1^2 and σ_2^2.

Fig.4. The same as in Figure 3 at U_0 = 2.

Fig.5. The dependence of the critical value of the parameter $\mu = m_2/m_1$ on the value of U_0 .

Fig.6. The ratio of the characteristic energies of heavy and light stars at the centre of the model as a function of U_0 at different values of μ .

Fig.7. Dashed line limits from below the range of the parameters (μ , U_0) wherein the deviation from equipartition exceeds 25%, i.e. ($m_2\sigma_2^2 - m_1\sigma_1^2$)/$m_1\sigma_1^2 \geq$ 0.25.

Fig.8. The ratio $m_2\sigma_2^2/m_1\sigma_1^2$ as a function of the coordinate x at U_0 = 5, λ = 3, μ = 3/2 and 3.

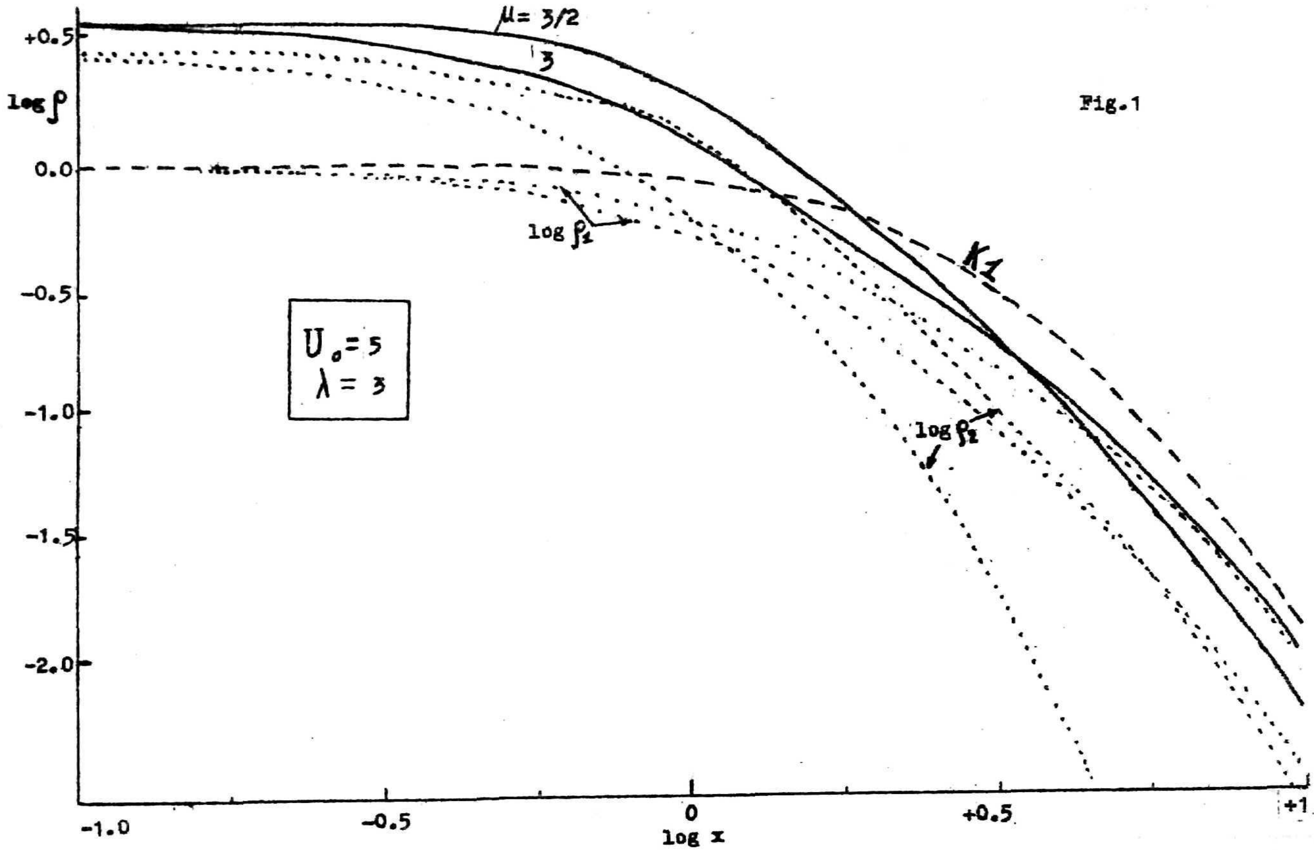

Fig.1

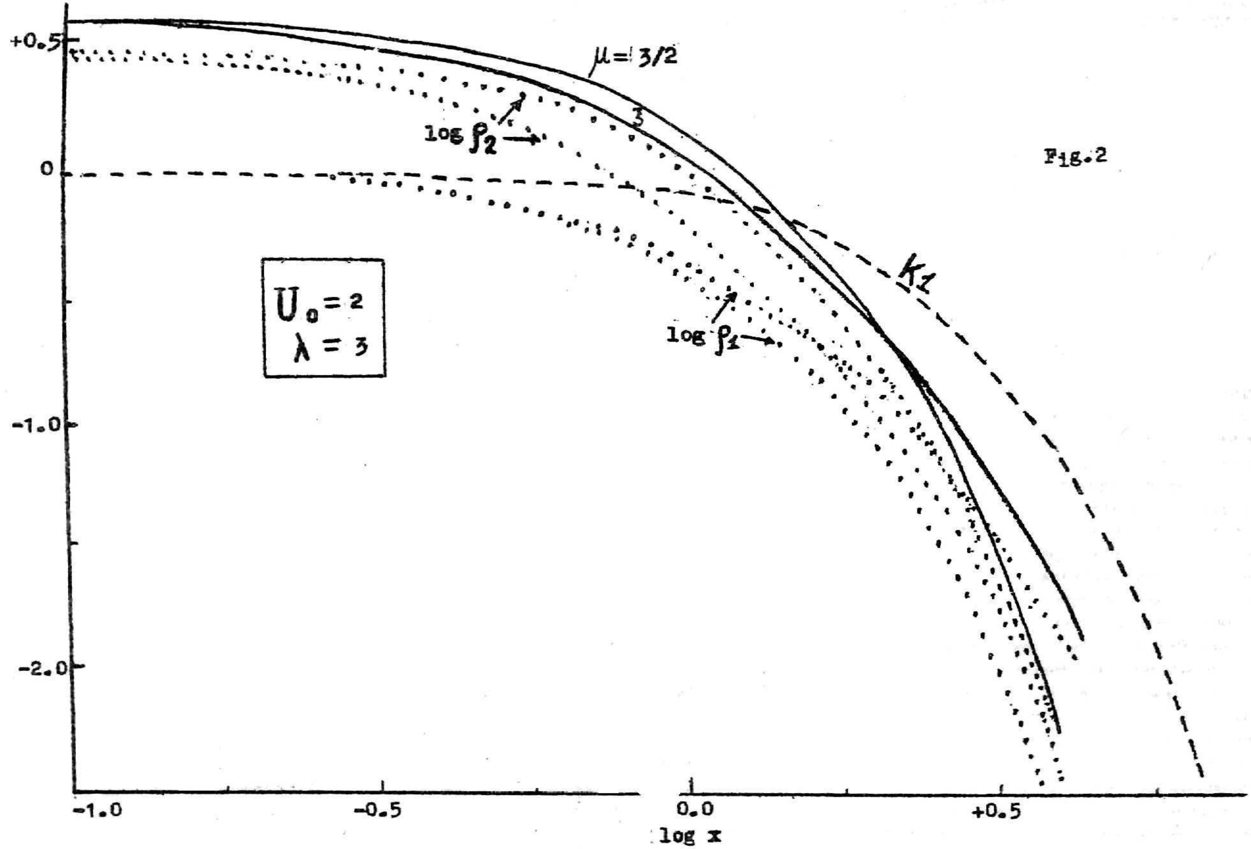
+0.5
0
-1.0
-2.0
-1.0
-0.5
0.0
+0.5
log x
$\mu = 3/2$
3
log ρ_2
log ρ_1
K_1
$U_0 = 2$
$\lambda = 3$

Fig.2

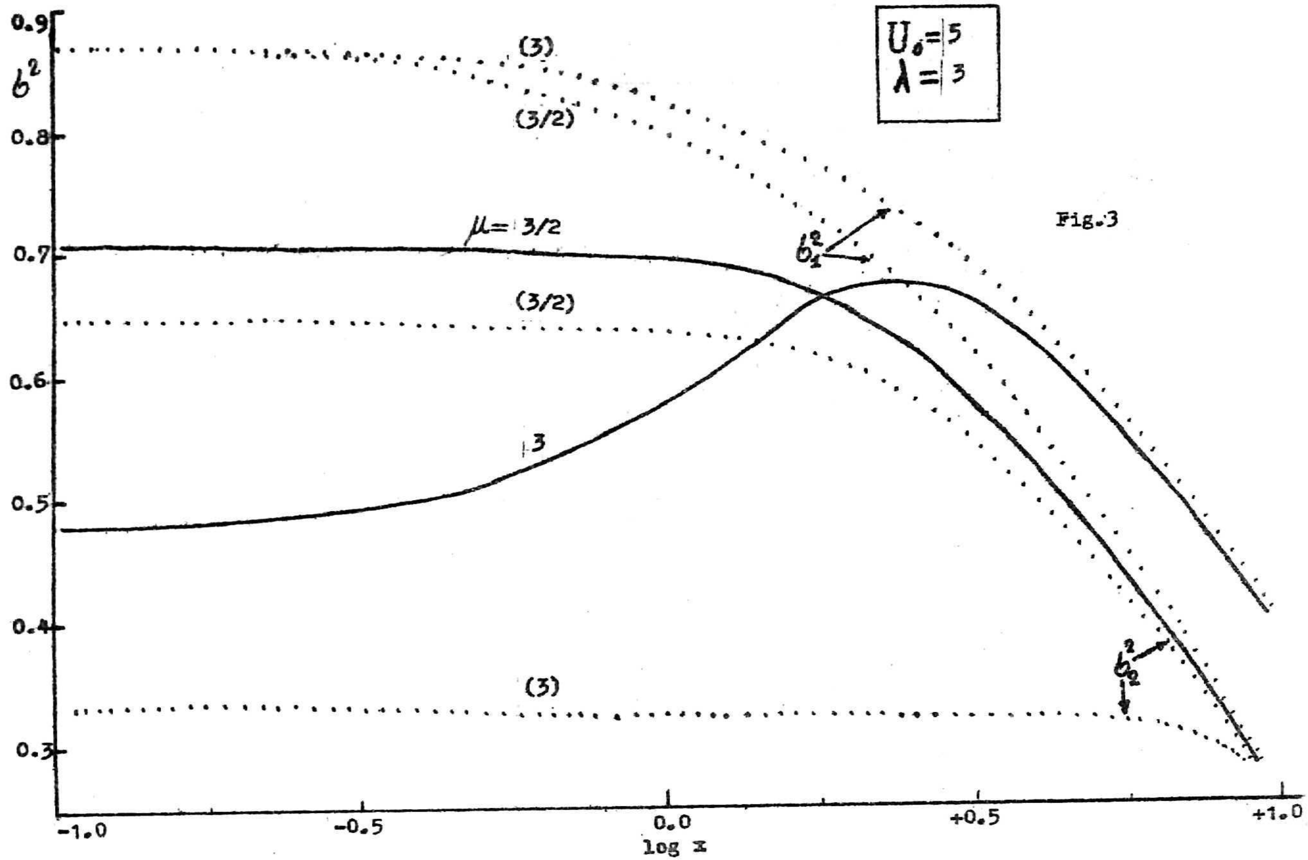
0.9
0.8
0.7
0.6
0.5
0.4
0.3
b^2
$U_0 = 5$
$\lambda = 3$
(3)
(3/2)
$\mu = 3/2$
(3/2)
3
(3)
b_1^2
b_2^2
-1.0
-0.5
0.0
+0.5
+1.0
log x

Fig.3

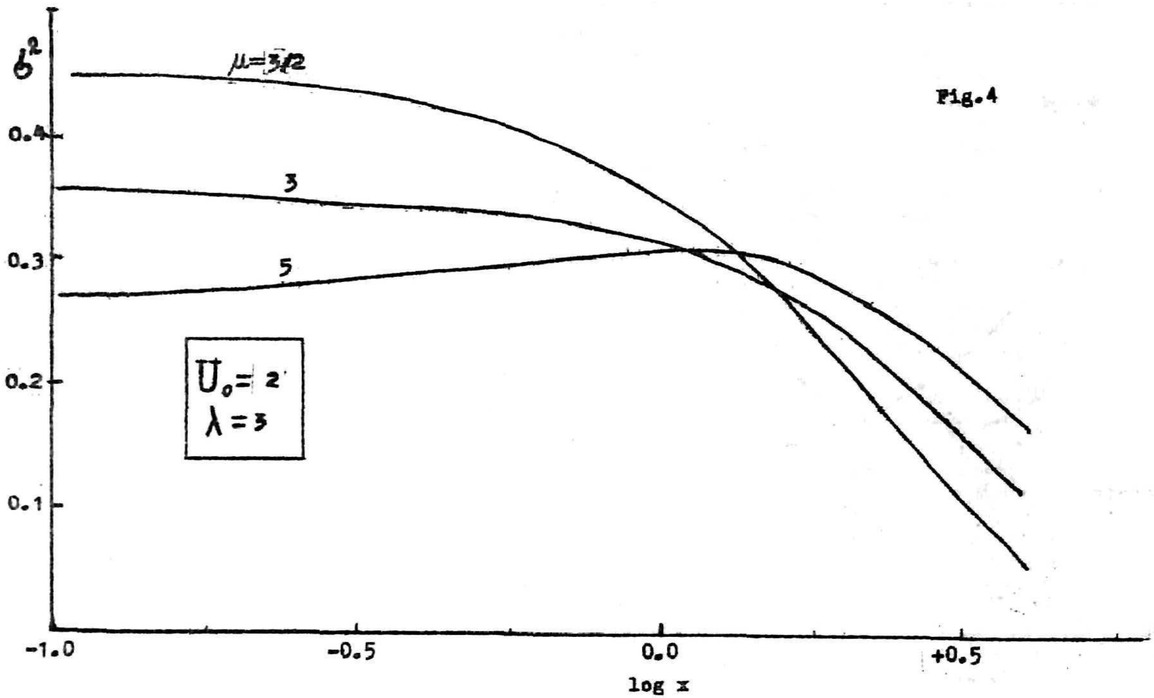

Fig.4

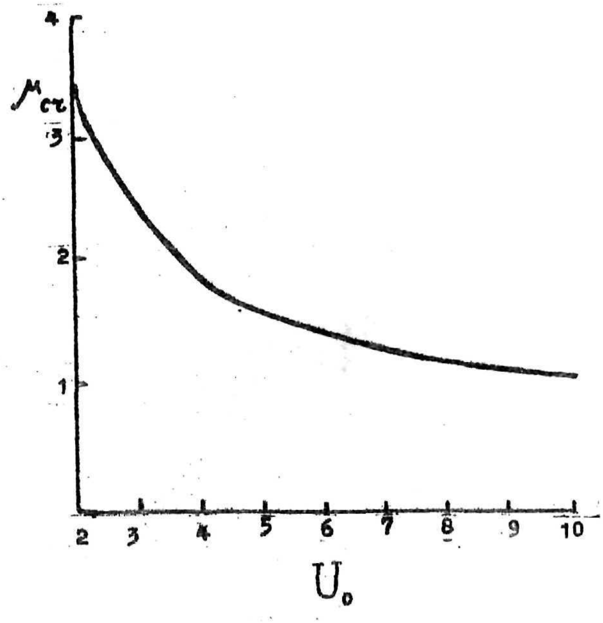

Fig.5

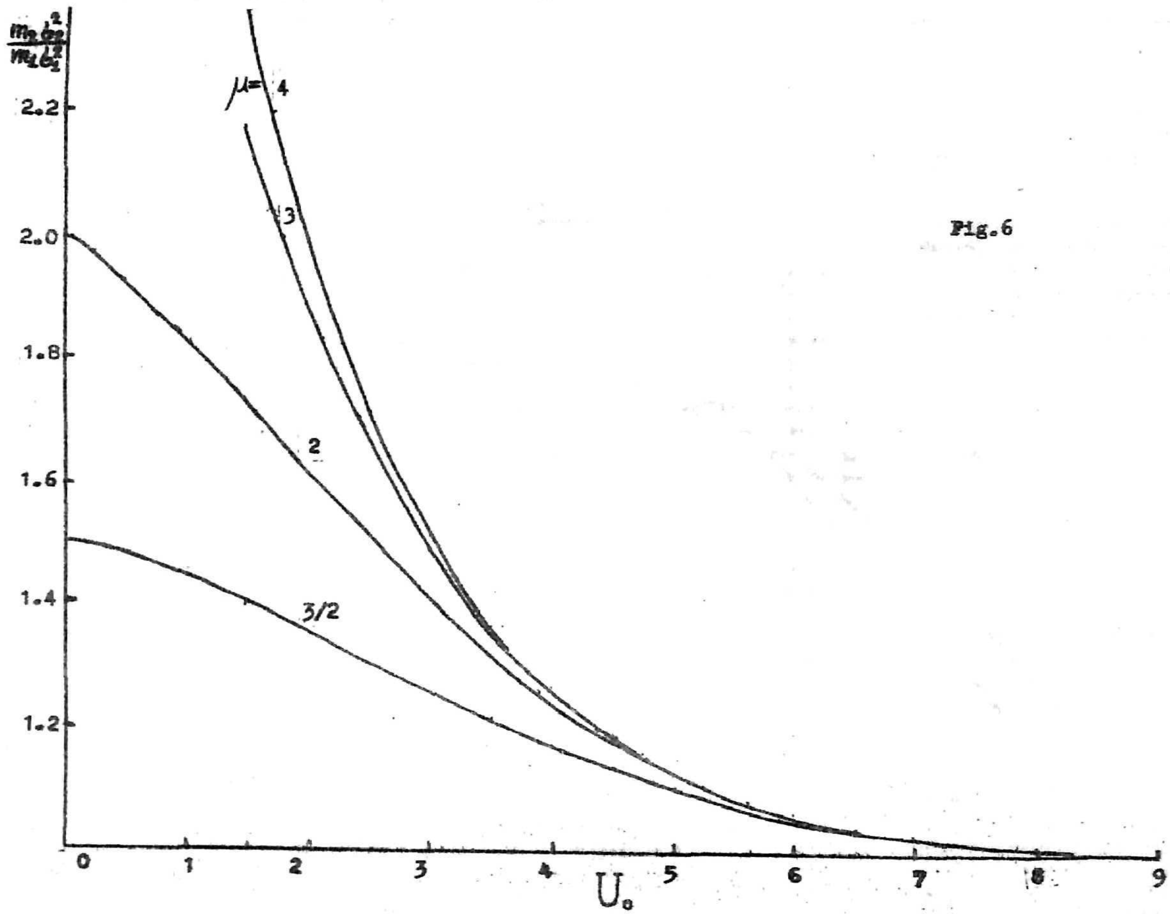

Fig.6

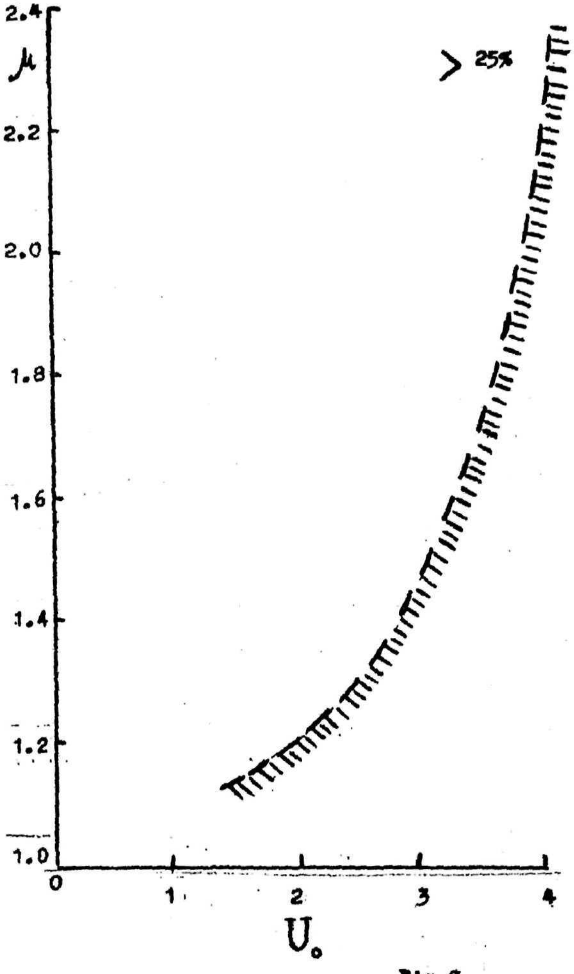

Fig.7

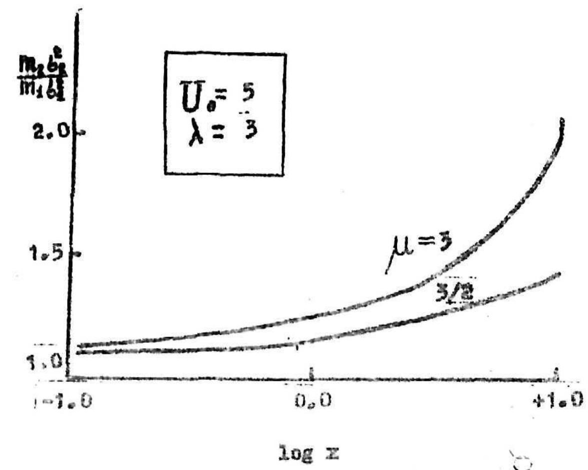

Fig.8

Т - 21452

Подписано в печать 22 июня 1981 года

Заказ № 296. Тираж 100 экз. п/л 1,5

Отпечатано в Редакционно-издательском

отделе ФИАН СССР

Москва, В-312, Ленинский проспект, 53