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CHRONICLE

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A. G. Dobryanskii (Moscow). *Decaying Solutions of a Certain Class of Linear Equations* (October 18, 1996).

We consider the equation

$$y'' - x^n y = 0, \tag{1}$$

where $n > 0$ is an integer, $0 \le x < +\infty$.

It is known that this equation has solutions $y(x)$ such that

$$\lim_{x\to+\infty} y(x) = 0; \tag{2}$$

moreover, two arbitrary solutions satisfying condition (2) are linearly dependent.

Theorem. *A function $y(x)$ is a nontrivial solution of Eq.* (1) *satisfying condition* (2) *if and only if*

$$y(0)/y'(0) = -\Gamma\left(\frac{n+3}{n+2}\right)(n+2)^{2/(n+2)}\Big/\Gamma\left(\frac{n+1}{n+2}\right),$$

where $\Gamma(x)$ is the Euler gamma function.

Note that Eq. (1) cannot be solved in elementary or known special functions. The proof is performed on the basis of expanding a solution in powers of x and estimates in [1, p. 21].

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A. Yu. Kolesov (Yaroslavl) and **N. Kh. Rozov** (Moscow). *Parametric Buffer Phenomenon in a Singularly Perturbed Nonlinear Telegraph Equation* (March 10, 2000).

On the closed interval $0 \le x \le \pi$, we consider the boundary value problem

$$u_{tt} + \varepsilon u_t + (1 + \varepsilon\alpha\cos 2\tau)u = \varepsilon a^2 u_{xx} + f(u, u_t), \qquad u_x|_{x=0} = u_x|_{x=\pi} = 0, \tag{1}$$

where $0 < \varepsilon \ll 1$, $\tau = (1 + \varepsilon\delta)t$, and the positive parameters α and a and a parameter δ of an arbitrary sign are of the order of unity. The Taylor expansion of the function $f(u, v) \in C^\infty$ at the origin contains terms of order ≥ 2. As the phase space [the space of initial conditions (u, u_t)] of problem (1), we take $\mathring{W}_2^2(0,\pi) \times W_2^1(0,\pi)$ and investigate the existence and stability (in the norm of this space) of solutions 2π-periodic in τ.

It was shown in [1] that the problem under consideration can be reduced to analyzing equilibria of the so-called quasinormal form of problem (1), which has the form

$$\xi_t = -\left(ia^2/2\right)\xi_{xx} - (1/2 + i\delta)\xi + (i\alpha/4)\bar{\xi} + d\xi|\xi|^2, \qquad \xi_x|_{x=0} = \xi_x|_{x=\pi} = 0, \tag{2}$$

where d is the complex Lyapunov quantity of the equation obtained from (1) for $\varepsilon = 0$, ξ is a complex function, and $\bar{\xi}$ satisfies the complex-conjugate equation. More precisely, each equilibrium

A. N. Vetokhin (Moscow). *On a Certain Property of the Upper Central Exponent* (March 31, 2000).

For a given positive integer n, we consider the space $\mathscr{M}_n^u$ of systems of the form

$$\dot{x} = A(t)x, \qquad x \in \mathbf{R}^n, \qquad t \in \mathbf{R}^+, \tag{1}$$

where $A : \mathbf{R}^+ \to \operatorname{End} \mathbf{R}^n$ is a continuous bounded operator function. This space is equipped with the topology of uniform convergence of coefficients on $\mathbf{R}^+$. Recall that the upper central exponent is given by the formula [1]

$$\Omega(A) = \lim_{T\to\infty} \overline{\lim_{m\to\infty}} \frac{1}{mT} \sum_{j=0}^{m-1} \ln |X(T(j+1), Tj)|,$$

where $X(t, s)$ is the Cauchy operator of system (1). It was proved in [2] that the Lyapunov upper central exponent $\Lambda(\cdot)$ treated as a function on the space $\mathscr{M}_n^u$ does not belong to the first Baire class. Therefore, we encounter the problem of finding the minimal function $\varphi(\cdot)$ from the first Baire class satisfying the condition $\Lambda(A) \le \varphi(A)$ for any system (1). This problem is solved in the following assertion.

Theorem. *Let the remainder functional $\varphi(\cdot) : \mathscr{M}_n \to \mathbf{R}$ belong to the first Baire class on the space $\mathscr{M}_n^u$ and satisfy the inequality $\Lambda(A) \le \varphi(A) \le \Omega(A)$ for any system* (1). *Then $\varphi(A) = \Omega(A)$ for any system* (1).

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1. Bylov, B.F., Vinograd, G.E., Grobman, D.M., and Nemytskii, V.V., *Teoriya pokazatelei Lyapunova i ee prilozheniya k voprosam ustoichivosti* (Theory of Lyapunov Exponents and Its Application to Stability Problems), Moscow, 1968.
2. Rakhimberdiev, M.I., *Mat. Zametki*, 1962, vol. 31, no. 6, pp. 925–931.

V. A. Zaitsev (Izhevsk). *On the Control for Lyapunov Characteristic Exponents of a Stationary System with Observer* (March 31, 2000).

We consider the following stationary linear system with observer:

$$\dot{x} = Ax + Bu, \qquad y = C^*x, \qquad (x, u, y) \in \mathbf{R}^{n+m+k}, \tag{1}$$

where $A = J_1 - \sum_{i=1}^n a_{n+1-i} e_n e_i^*$, $J_p = \{g_{ij}\}_{i,j=1}^n$, $g_{i,i+p} = 1$, $i = 1, \dots, n-p$, and $g_{ij} = 0$ for $j - i \ne p$ $(0 \le p \le n-1)$. The control is constructed in the form $u = Vy$, which implies the system $\dot{x} = (A + BVC^*)\, x$.

Definition. System (1) has the property of *global controllability of Lyapunov exponents* if for any $\gamma = (\gamma_1, \dots, \gamma_n) \in \mathbf{R}^n$, there exists a matrix V such that $\chi\,(A + BVC^*, \lambda) = \lambda^n + \gamma_1\lambda^{n-1} + \dots + \gamma_n$, where $\chi(A, \lambda)$ is the characteristic polynomial of the matrix A.

We construct the matrix $G = \sum_{i=1}^n a_{i-1} J_{i-1}^*$, $a_0 = 1$. Let an $n \times n$ matrix D have the block form $\begin{pmatrix} 0 & 0 \\ F & 0 \end{pmatrix}$, where F is an $(n-p+1) \times p$ matrix, $p \in \{1, \dots, n\}$.

Theorem 1. *Let $\chi(A+D, \lambda) = \lambda^n + \gamma_1\lambda^{n-1} + \dots + \gamma_n$. Then $\gamma_i = a_i - \operatorname{Tr} DJ_{i-1}G$, $i = 1, \dots, n$.*

Theorem 2. *Let $C^* e_i e_j^* B = 0$ for all $1 \le j < i \le n$. System* (1) *has the property of global controllability of Lyapunov exponents if and only if $C^* J_{i-1} GB$, $i = 1, \dots, n$, are linearly independent matrices. In this case, the matrix V reducing the characteristic polynomial of the matrix $A + BVC^*$ to a predefined polynomial $\lambda^n + \gamma_1\lambda^{n-1} + \dots + \gamma_n$ has the form $V = \left[\operatorname{vec}^{-1}\left(P\,(P^*P)^{-1}\,(a-\gamma)\right)\right]^*$, where $P = [\operatorname{vec} C^* J_0 GB, \dots, \operatorname{vec} C^* J_{n-1} GB]$ is an $mk \times n$ matrix, $a = (a_1, \dots, a_n) \in \mathbf{R}^n$, and $\operatorname{vec} H$ is the operator "extracting" the matrix H by rows in a vector column.*

Example. Let us consider the following linear nth-order equation with an observer:

$$\begin{aligned} z^{(n)} &+ a_1 z^{(n-1)} + a_2 z^{(n-2)} + \cdots + a_n z \\ &= \beta_{p1} u_1^{(n-p)} + \beta_{p+1,1} u_1^{(n-p-1)} + \cdots + \beta_{n1} u_1 + \cdots + \beta_{pm} u_m^{(n-p)} \\ &+ \cdots + \beta_{nm} u_m, \qquad z \in \mathbf{R}, \qquad 1 \le p \le n, \end{aligned} \tag{2}$$

$$y_1 = c_{11} z + \cdots + c_{p1} z^{(p-1)}, \qquad \ldots, \qquad y_k = c_{1k} z + \cdots + c_{pk} z^{(p-1)}, \tag{3}$$

$u = (u_1, \ldots, u_m) \in \mathbf{R}^m$, $y = (y_1, \ldots, y_k) \in \mathbf{R}^k$. Let $u = Vy$. On the basis of system (2), (3), we construct an $n \times n$ matrix A, an $n \times m$ matrix $K = \{\beta_{ij}\}$ in which the first $p - 1$ rows are zero, and an $n \times k$ matrix $C = \{c_{ij}\}$ in which the last $n - p$ rows are zero. Then system (2), (3) with an observer is equivalent to the matrix system (1) with the matrix $B = G^{-1}K$, where the correspondence is given by the relation $x_1 = z$. The assumptions of Theorem 2 are valid. Therefore, if $C^* J_{i-1} K$, $i = 1, \ldots, n$, are linearly independent matrices, then for any $\gamma = (\gamma_1, \ldots, \gamma_n) \in \mathbf{R}^n$ the control $V = \left[\operatorname{vec}^{-1}\left(P\left(P^*P\right)^{-1}(a - \gamma)\right)\right]^*$, where $P = [\operatorname{vec} C^* J_0 K, \ldots, \operatorname{vec} C^* J_{n-1} K]$, reduces system (2), (3) to the equation $z^{(n)} + \gamma_1 z^{(n-1)} + \gamma_2 z^{(n-2)} + \cdots + \gamma_n z = 0$.

V. S. Samovol (Moscow). *Transformations of Invertible Systems* (April 7, 2000).

We consider the problem on the possibility to reduce a real system of ordinary differential equations

$$\dot{x} = Ax + F(x), \qquad \|F(x)\| = o(\|x\|), \tag{1}$$

of the class C^∞ in a neighborhood of a nondegenerate saddle singular point of the space $\mathbf{R}^n$ with the use of a nondegenerate invertible diffeomorphism

$$x = H(y) \tag{2}$$

of class C^k $(k \ge 1)$ either to the normal form

$$\dot{y} = P(y) \tag{3}$$

(the local C^k-normalization), where $P(y)$ is a polynomial consisting of resonance terms, or to the linear form

$$\dot{y} = Ay \tag{4}$$

(the local C^k-linearization).

It is of interest to consider the case in which system (1) is invertible, i.e., there exists a nondegenerate matrix B such that

$$AB = -BA, \qquad F(Bx) = -BF(x).$$

In this case, B is referred to as the automorphism matrix of system (1). It is important to preserve the invertibility under the transformation (2), which is provided by the condition

$$H(By) = BH(y). \tag{5}$$

It is known that problem of local smooth normalization of system (1) can be solved with the use of the Sternberg–Chen theorem [1]. A condition imposed on the normal form of system (1) and sufficient for its local C^k-linearization was given in [2] [the condition $S(k)$].

Theorem 1. *If the automorphism matrix B of system* (1) *satisfies the condition*

$$B^m = E, \tag{6}$$

where E is the identity matrix and $m > 0$ is an integer, then for system (1), *there exists a transformation* (2) *satisfying condition* (5) *and reducing it to the normal form* (3).