are satisfied almost surely for some $h > 0$. We denote the random variables (3) by $c_i(\omega)$, where $i = [t/h]$, represent their dispersions in the form $\sigma_i = d_i/\ln i$, and consider the series

$$\sum_{i=1}^{\infty} \frac{d_i}{i^{A^2/d_i^2} \ln i}. \tag{4}$$

Let κ_g and κ_g' be the upper and lower general exponents [1] of Eq. (1), and let $\kappa_g(\omega)$ and $\kappa_g'(\omega)$ be those of Eq. (2).

Theorem. *The relations*

$$\kappa_g(\omega) = \kappa_g, \qquad \kappa_g'(\omega) = \kappa_g' \tag{5}$$

are valid almost surely if and only if there exists a number A such that the series (4) *is convergent.*

Corollary 1. *If the sequence d_i is uniformly bounded above, then relations* (5) *hold almost surely.*

Corollary 2. *If $d_i \to \infty$ $(i \to \infty)$, then the relations*

$$\kappa_g(\omega) = \infty, \qquad \kappa_g'(\omega) = -\infty$$

hold almost surely.

Corollary 3. *There exist admissible random perturbations* [2] *of Eq.* (1) *under which the general exponents are unstable.*

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V. A. Zaitsev and E. L. Tonkov (Izhevsk, Russia). *Uniform Exponential Stabilization of a Family of Control Systems* (March 18, 2011).

The report deals with the discussion of results of [1]. Let (Σ, h^t) be a topological dynamical system with compact phase space Σ, and let $f(\sigma, x, u)$ be a given function of the variables $(\sigma, x, u) \in \Sigma \times \mathbb{R}^n \times \mathbb{R}^m$. Next, suppose that, for each $\sigma \in \Sigma$, the function $f(h^t\sigma, x, u)$ of the variables (t, x, u) is piecewise continuous with respect to t and has continuous partial derivatives with respect to the variables x and u on the set $(t, x, u) \in \mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^m$.

Consider the family of control systems

$$\dot{x} = f(h^t\sigma, x, u), \qquad (t, x, u) \in \mathbb{R} \times \mathbb{R}^n \times U, \qquad U \subseteq \mathbb{R}^m, \tag{1}$$

with parameter $\sigma \in \Sigma$, where U is a given set in $\mathbb{R}^m$. Let us recall the following notion.

Definition 1. A function $t \mapsto \widehat{\varphi}(t, \sigma) := (\widehat{x}(t, \sigma), \widehat{u}(t, \sigma)) \in \mathbb{R}^n \times U$ is referred to as an *admissible process* of system (1) if it is defined and bounded on the set $\mathbb{R}$, the function $t \mapsto \widehat{u}(t, \sigma)$ is Lebesgue integrable on each closed interval of the line $\mathbb{R}$, and the function $t \mapsto \widehat{x}(t, \sigma)$ is a Carathéodory solution (on the entire line $\mathbb{R}$) of the system of equations

$$\dot{x} = f(h^t\sigma, x, \widehat{u}(t, \sigma)). \tag{2}$$

Definition 2. An admissible process $\widehat{\varphi}(t, \sigma)$ of system (1) is said to be *uniformly exponentially stabilizable with exponent* $\alpha > 0$, if there exist numbers $N > 0$, $\gamma > 0$, and $\delta > 0$ independent of σ (but not necessarily independent of α) and such that, for any $\sigma \in \Sigma$, there exists a control $u(t,x,\sigma)$ locally Lebesgue integrable with respect to t, continuous with respect to x, and satisfying the following three conditions for all $(t,x) \in \mathbb{R} \times O_\delta(\widehat{x}(t,\sigma))$.

1. $u(t,\widehat{x}(t,\sigma),\sigma) = \widehat{u}(t,\sigma)$.
2. The inequality $|u(t,x,\sigma) - \widehat{u}(t,\sigma)| \le \gamma$ holds.
3. Every solution $t \mapsto x(t,\sigma)$ of system (1) with the chosen control $u = u(t,x,\sigma)$ and with the initial condition $x(0,\sigma) \in O_\delta(\widehat{x}(0,\sigma))$ satisfies the inequality

$$|x(t,\sigma) - \widehat{x}(t,\sigma)| \le Ne^{-\alpha t}$$

for all $t \ge 0$.

For an admissible process $\widehat{\varphi}(t,\sigma)$ and system (2), we construct the *first approximation system*

$$\dot{y} = A(t,\sigma)y + B(t,\sigma)v, \quad A(t,\sigma) := \left.\frac{\partial f(h^t\sigma,x,u)}{\partial x}\right|_{\widehat{\varphi}(t,\sigma)}, \quad B(t,\sigma) := \left.\frac{\partial f(h^t\sigma,x,u)}{\partial u}\right|_{\widehat{\varphi}(t,\sigma)}. \tag{3}$$

Recall [2] that system (3) is said to be *uniformly completely controllable* (*on* Σ) if there exist numbers $\vartheta, \ell > 0$ such that, for any point $(y_0,\sigma) \in \mathbb{R}^n \times \Sigma$, there exists a control $v(t,y_0,\sigma)$ measurable with respect to $t \in [0,\vartheta]$, continuous with respect to (y_0,σ), bringing a solution $y(t,\sigma)$ of Eq. (3) from the point $y(0,\sigma) = y_0$ to the point $y(\vartheta,\sigma) = 0$, and such that the inequality $|v(t,y_0,\sigma)| \le \ell|y_0|$ holds for all $t \in [0,\vartheta]$.

By using the Millionshchikov rotation method and by generalizing the Lyapunov, Malkin, and Massera theorems on the stability of a system with respect to the first approximation, we obtain an algorithm for constructing a control solving the stabilization problem for a given admissible process of the control system (1).

Theorem. *Let an admissible process* $\widehat{\varphi}(t,\sigma)$ *of system* (1) *and a constant* $\alpha > 0$ *be given. If the function* $(x,u) \mapsto f(\sigma,x,u)$ *is twice continuously differentiable and system* (3) *is uniformly completely controllable*, *then there exists a control* $u(t,x,\sigma)$ *solving the problem of uniform exponential stabilization of the process* $\widehat{\varphi}(t,\sigma)$ *with exponent* $\alpha > 0$.

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L. I. Rodina (Izhevsk, Russia). *Statistically Invariant*, *with Probability* 1, *Sets of Control Systems with Random Parameters* (March 25, 2011).

We use the results in [1, 2] and consider the control problem given by a metric dynamical system $(\Sigma, \mathfrak{A}, \nu, h^t)$, the control system

$$\dot{x} = f(h^t\sigma,x,u), \qquad (t,\sigma,x,u) \in \mathbb{R} \times \Sigma \times \mathbb{R}^n \times \mathbb{R}^m, \tag{1}$$

defining the dynamics of the process, a set M containing the additional constraints of the problem, and a set U in $\mathbb{R}^m$ that defines geometric constraints for admissible controls. In a number of control problems, it is admissible to violate given constraints temporarily for the set of admissible controls, but in general, the control should performed so as to ensure that the relative frequency of the trajectory being in a given set is equal to unity. In the present paper, we consider conditions under which the given set M is statistically invariant with probability 1.

We assume that there exists a set $\Sigma_0 \subseteq \Sigma$ such that $\nu(\Sigma_0) = 1$ and the following conditions are satisfied for each $\sigma \in \Sigma_0$.

(a) $(x,u) \mapsto f(h^t\sigma,x,u)$ is a continuous function.

(b) $t \mapsto f(h^t\sigma,x,u)$ is a piecewise continuous function.

(c) The function $(t,x) \mapsto U(h^t\sigma,x) \in \mathrm{comp}(\mathbb{R}^m)$ is upper semicontinuous for all $(t,x) \in \mathbb{R}^{n+1}$.