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PROBLEMS OF IMPULSIVE AND MIXED CONTROL FOR LINEAR FUNCTIONAL DIFFERENTIAL SYSTEMS[1]

An approach to the study of differential equations with discontinuous solutions is associated with the so called "generalized ordinary differential equations"whose theory was initiated by J.Kurzweil. Nowadays this theory is highly developed. According to the accepted approaches impulsive equations are considered within the class of functions of bounded variation. In this case the solution is understood as a function of bounded variation satisfying an integral equation with the Lebesgue-Stiltjes integral or Perron-Stiltjes one. Recall that the function of bounded variation is representable in the form of the sum of an absolutely continuous function, a break function, and a singular component (a continuous function with the derivative being equal zero almost everywhere). The solutions of equations with impulse impact, which are considered below, do not contain the singular component and may have discontinuity only at finite number of prescribed points. Following [1, 2] we consider these equations on a finite-dimensional extension $\mathbf{DS}(m)$ (see below) of the traditional space $\mathbf{AC}$ of absolutely continuous functions. This approach to the equations with impulsive impact was offered in [2]. It does not use the complicated theory of generalized functions, turned out to be rich in content and finds many applications in the cases where the question about the singular component does not arise.

Let $\mathbf{L}$ be the Banach space of Lebesgue summable functions $z : [0,T] \to \mathbf{R^n}$, $||z||_{\mathbf{L}} = \int_0^T ||z(s)||\,ds$, let us fix a collection of points $t_k \in (0,T)$, $0 < t_1 < ... < t_m < T$. Consider the space $\mathbf{DS}(m)$ of functions $x : [0,T] \to \mathbf{R^n}$ that are representable in the form

$$x(t) \;=\; \int_0^t z(s)\,ds \;+\; x(0) \;+\; \sum_{k=1}^{m} \chi_{[t_k,T]}(t)\Delta x(t_k),$$

where $z \in \mathbf{L}$, $\Delta x(t_k) = x(t_k) - x(t_k - 0)$, $\chi_{[t_k,T]}(t)$ is the characteristic function of the segment $[t_k, T]$. The norm in $\mathbf{DS}(m)$ is defined by $||x||_{\mathbf{DS}(m)} = ||z||_{\mathbf{L}} + ||\{x(0), \Delta x(t_1), ..., \Delta x(t_m)\}||$.

Consider the functional differential system

$$(\mathcal{L}x)(t) \;=\; f(t), \;\; t \in [0,T], \tag{1}$$

where $\mathcal{L} : \mathbf{DS}(m) \to \mathbf{L}$ is linear bounded and has the principal part of the form

$$(Qz)(t) \;=\; z(t) \;-\; \int_0^t K(t,s)z(s)\,ds.$$

Here the elements $k^{ij}(t,s)$ of the kernel $K(t,s)$ are measurable on the set $0 \leqslant s \leqslant t \leqslant T$ and such that $|k^{ij}(t,s)| \leqslant \mu(t)$, $i,j = 1,...,n$, $\mu(\cdot)$ is summable on $[0,T]$. Notice that the form of (1) covers many classes of dynamic models including differential systems with distributed or/and concentrated delay and integro-differential systems.

The space of all solutions to the homogenous system $\mathcal{L}x = 0$ is finite-dimensional, its dimension equals $n + nm$. Let $\{x_1, ..., x_{n+nm}\}$ be a basis in this space. The matrix $X = \{x_1, ..., x_{n+nm}\}$ is called a fundamental matrix. The general solution of (1) is representable in the form

$$x(t) \;=\; X(t)\cdot\sigma \;+\; \int_0^t C(t,s)f(s)\,ds, \tag{2}$$

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where $C(t,s)$ is the Cauchy matrix, $\sigma \in \mathbf{R^{n+nm}}$. Let $l : \mathbf{DS}(m) \to \mathbf{R^N}$ be a linear bounded vector-functional. There takes place the representation

$$lx = \int_0^T \Phi(s)\dot{x}(s)\,ds + \Psi_0 x(0) + \sum_{k=1}^{m} \Psi_k \Delta x(t_k), \tag{3}$$

where the elements of measurable $N \times n$-matrix Φ are essentially bounded and Ψ_k, $k = 0, ..., m$, are $N \times n$-matrices with real elements.

Consider the control problem

$$\mathcal{L}x = Fu + f, \;\; x(0) = \alpha, \; lx = \beta. \tag{4}$$

Here $F : \mathbf{L_2} \to \mathbf{L}$ is a given linear bounded operator, $\mathbf{L_2}$ is the space of square summable functions $u : [0,T] \to \mathbf{R^r}$ with inner product $(u,v) = \int_0^T u^\top(s)v(s)\,ds$, $\cdot^\top$ is the symbol of transposition. In the problem (4), the target of controlling is defined by the vector-functional $l : \mathbf{DS}(m) \to \mathbf{R^N}$ whose value on a trajectory of $\mathcal{L}x = Fu + f$ must reach (under control) the vector $\beta \in \mathbf{R^N}$. The problem (4) includes in particular the control problem with L_2-control (the case that the condition $lx = \beta$ includes the equalities $\Delta x(t_k) = 0$, $k = 1, ..., m$) and the control problem with only impulse control (the case $F = 0$, where the role of control actions is played only by the jumps $\Delta x^i(t_k)$). The latter case is considered in details in [3]. Here we give conditions of controllability through the mixed control.

Let us denote

$$\Theta(s) = \Phi(s) + \int_s^T \Phi(s) C'_\tau(\tau, s)\,d\tau,$$

$$\Xi = \int_s^T \Phi(s)\dot{X}(s)\,ds = (\,\Xi_1 \,|\, \Xi_2\,),$$

where Ξ_1 is the $N \times n$-matrix consisting of the first n columns of $N \times (n + nm)$-matrix Ξ;

$$M = \int_0^T [F^*\Theta](s)[F^*\Theta]^\top(s)\,ds,$$

where $F^* : \mathbf{L}^* \to \mathbf{L_2^*}$ is the adjoint operator to F.

T h e o r e m 1. *The control problem* (4) *is solvable if and only if the linear algebraic system*

$$[\Xi_2 + (\Psi_1, ..., \Psi_m)] \cdot \delta + M \cdot \gamma = \tag{5}$$

$$\beta - \int_0^T \Theta(s)f(s)\,ds - (\Xi_1 + \Psi_1 + \Psi_0) \cdot \alpha$$

is solvable in $(N + nm)$*-vector* $col(\delta, \gamma)$. *Each solution* $col(\delta_0, \gamma_0)$, $\delta_0 = col(\delta_0^1, ..., \delta_0^m)$, *of the system* (5) *defines the control that solves control problem* (4) : $\Delta x(t_k) = \delta_0^k, k = 1, ..., m,$ $u(t) = [F^*\Theta]^\top(t) \cdot \gamma_0$.

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Vladimir P. Maksimov
Perm State University,
Russia, Perm
e-mail: maksimov@econ.psu.ru