УДК 517.929; 519.863.6; 519.863.2

© **P. M. Simonov**

ON A METHOD OF RESEARCH OF DYNAMIC ECONOMIC MODELS [1]

A new approach to research of differential equations with aftereffect is considered in the report. Modifications of some well-known models micro- and macroeconomics are offered. Effective sufficient stability conditions of such models are demonstrated.

In the last 50 years models with delay (with lag, with aftereffect) are applied in all sciences: from mechanics to sociology. These models are more adequate for miscellaneous processes of reality (see, e.g. [1]–[3]). One of central problems is the problem of the correctness or the stability of models (see, e.g. [1], [2]). Academician N.N. Krasovskii has elaborated and has extended to the given subjects Lyapunov's second method [4]. In our research an other approach is offered. This approach has received the title a «method of elementary models» (a «method of model equations»). Ideas of the method may be found in classical publications of J. Liouville as well as in some works of G. Fubini and F.G. Tricomi. Mathematicians from Perm (N.V. Azbelev and his colleagues) offered the investigation method based on a reduction of a differential equation to an operator equation of the second kind [5]–[7]. As a result, it has become possible to study the linear differential equation with delay by methods of the linear functional analysis. The new approach allowed one to reduce both differential equations with delay and boundary value problems for them. Subsequently these methods of transformation and the research of differential equations were called as the $W-$ method of Azbelev. Then this idea has been applied to the problem of stability [8], [9].

In our report we describe some models of microeconomics and macroeconomics [10], [11]. The elementary models method is applicable to these models [12]. All of these have the inertial delay with the finite transient period. We offered the model of such time delay between an input process and an output process in the form of the linear differential equation with piecewise constant argument (step argument)

$$T\dot{y}(t) + y([t/T]) = x(t), \quad t \geqslant 0,$$

where $x(t)$ is an input process, $y(t)$ is an output process, $[\alpha]$ is the integer part of a number α . In all models such inertial delay with piecewise constant argument (IDPCA) [10], [11] was taken into account. In some models there are also other new features. We shall consider in detail some models of macroeconomics. For example, in the neoclassical nonlinear one-sector model of F.P. Ramsey, R.M. Solow, and T. Swan (RSS) of dynamics of the gross domestic product (GDP) there is used the idea by the paper of V.E. Nakoryakov, and V.G. Gasenko [13] about the separation on the investments in the fixed assets (in the fixed capital) and the induced investment in the current assets (in the working capital). Last investments are determined by the link of the GDP accelerator. In the neoclassical nonlinear model of W.-B. Zhang [14] of the GDP dynamics there is used the IDPCA between an input of the investments and the formation of the human capital. In the neoclassical nonlinear model of H. Uzawa and R.E. Lucas (jr.) [15], [16] for the dynamics of the GDP and the human capital are entered the similar delays. In the neoclassical nonlinear one-sector model of J. Tobin and M. Sidrauski [14], [17] of the GDP dynamics with regard for the money market there are considered the static and the adaptive expectations of inflation, including the adaptive expectations of inflation of IDPCA type. In all models the general production functions are used.

List of literature

1. Kolmanovskii V.B., Nosov V.R. Stability of functional differential equations. London e.a.: Academic Press, Inc. 1986.

2. Hale J.K., Lunel S.M.V. Introduction to functional differential equations. New York e.a.: Springer Verlag, 1993.

3. Kolmanovskii V.B., Myshkis A.D. Introduction to the theory and applications of functional differential equations. Dordrecht e. a.: Kluwer Academic Publishers, 1999.

4. Krasovskii N.N. Stability of motion: applications of Lyapunov's second method in differential systems of equations with delay. Stanford: Stanford Univ. Press, 1963.

5. Azbelev N.V., Maksimov V.P., Rakhmatullina L.F. Introduction to the theory of linear functional differential equations. Atlanta: World Federation Publishers Company, 1995.

6. Azbelev N.V., Rakhmatullina L.F. Theory of linear abstract functional differential equations and applications // Memoirs on Different. Equat. and Math. Physics. Tbilisi: Publishing House GCI, 1996. V. 8. P. 1–102.

7. Azbelev N.V., Maksimov V.P., Rakhmatullina L.F. The elements of the contemporary theory of functional differential equations. Methods and applications. Moscow: Institute of computer-assisted research, 2002 (in Russian).

8. Azbelev N.V., Simonov P.M. Stability of solutions of equations with ordinary derivatives. Perm: Perm University Publishung, 2001 (in Russian).

9. Azbelev N.V., Simonov P.M. Stability of differential equations with aftereffect. London and New York: Taylor and Francis, 2002.

10. Simonov P.M. On some dynamical models of microeconomics // Vestnik PGTU. Matematika i prikl. matematika / Perm State Technical University. Perm, 2002. P. 109–114 (in Russian).

11. Simonov P.M. On some dynamical models of macroeconomics // Econom. kibernetika: Mat. i instrum. metody analisa, prognozir. i upravl. Coll. of articles / Perm State University. Perm, 2002. P. 213–231 (in Russian).

12. Simonov P.M. Research of stability of solution of some dynamical micro- and macroeconomic models // Perm University Herald. Mathematics. Information Science. Mechanics. Scientific Journal. Issue 5 / Perm State University. Perm, 2003. P. 88–93 (in Russian).

13. Nakaryakov V.E., Gasenko V.G. A mathematical model of planned economy // Economika i mat. metody. 2002. V. 38, № 2. P. 118–124 (in Russian).

14. Zhang W.-B. Synergetic economics. Time and change in nonlinear economics. Berlin e.a.: Springer-Verlag, 1991.

15. Uzawa H. Optimum technical change in an aggregative model of economic growth // Internat. Economic Review. 1965. V. 6, № 1. P. 18–31.

16. Lucas R.E., Jr. On the mechanism of economic development // J. of Monetary Economics. 1988. V. 22, № 7. P. 3–42.

17. Harris L. Monetary. New York e.a.: McGraw-Hill Book Company, 1981.

Simonov Pyotr Mikhailovich
Perm State University,
Russia, Perm
e-mail: simonov@econ.psu.ru